11.03.26

4.3.6 Résoudre les équations suivantes.

b) $\begin{cases} \sin(t) + 3\cos(t) = 3 \\ \sin^2(t) + \cos^2(t) = 1 \end{cases} \Rightarrow \begin{cases} \sin(t) = 3 - 3\cos(t) \\ (3 - 3\cos(t))^2 + \cos^2(t) = 1 \end{cases}$

On résout la deuxième équation :

$$9 - \underline{18\cos(t)} + \underline{9\cos^2(t)} + \underline{\cos^2(t)} - 1 = 0$$

$$10\cos^2(t) - 18\cos(t) + 8 = 0$$

$$5\cos^2(t) - 9\cos(t) + 4 = 0$$

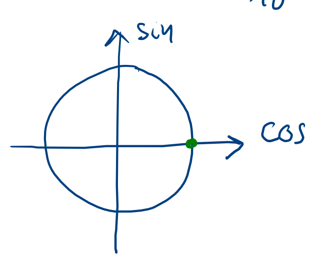
Posons $x = \cos(t)$ $\quad -1 \leq x \leq 1$

$$5x^2 - 9x + 4 = 0$$

$$\Delta = 81 - 80 = 1$$

$$x_1 = \frac{9+1}{10} = 1 \quad ; \quad x_2 = \frac{9-1}{10} = \frac{8}{10} = \frac{4}{5}$$

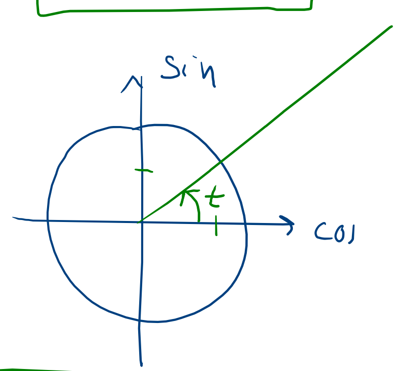
① $\begin{cases} \cos(t) = 1 \\ \sin(t) = 0 \end{cases}$



$\boxed{t = k \cdot 360°}$

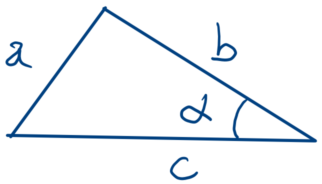
② $\begin{cases} \cos(t) = \frac{4}{5} \\ \sin(t) = 3 - 3 \cdot \frac{4}{5} = \frac{15-12}{5} = \frac{3}{5} \end{cases}$

$\boxed{t \overset{TI}{\cong} 36{,}87°}$

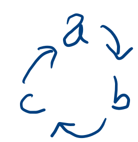


$\boxed{t = 36{,}87° + k \cdot 360°}$

Théorème du cosinus

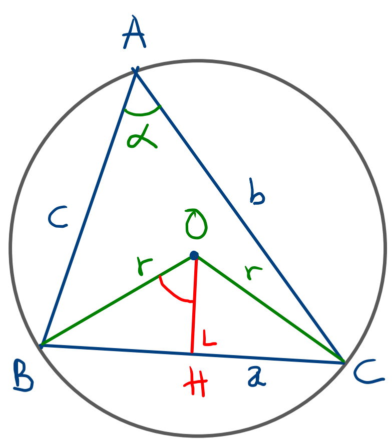


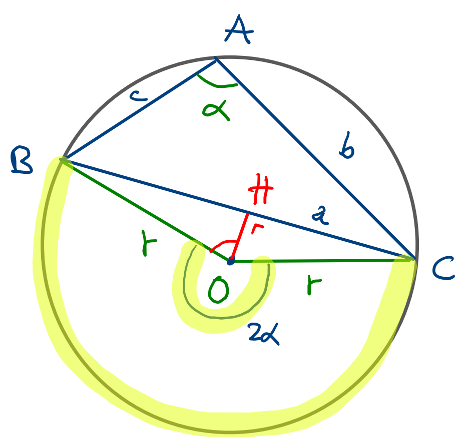
$$\boxed{a^2 = b^2 + c^2 - 2bc\cos(\alpha)}$$



Théorème du sinus

Soit un ΔABC et son cercle circonscrit de rayon r et de centre O





$$\widehat{BOC} = 2\alpha$$

$$\widehat{BOH} = \alpha$$

$$\sin(\alpha) = \frac{\frac{a}{2}}{r} = \frac{a}{2r}$$

$$2r = \frac{a}{\sin(\alpha)}$$

$$\widehat{BOC} = 360° - 2\alpha$$

$$\widehat{BOH} = 180° - \alpha$$

$$\underbrace{\sin(180° - \alpha)} = \frac{\frac{a}{2}}{r}$$

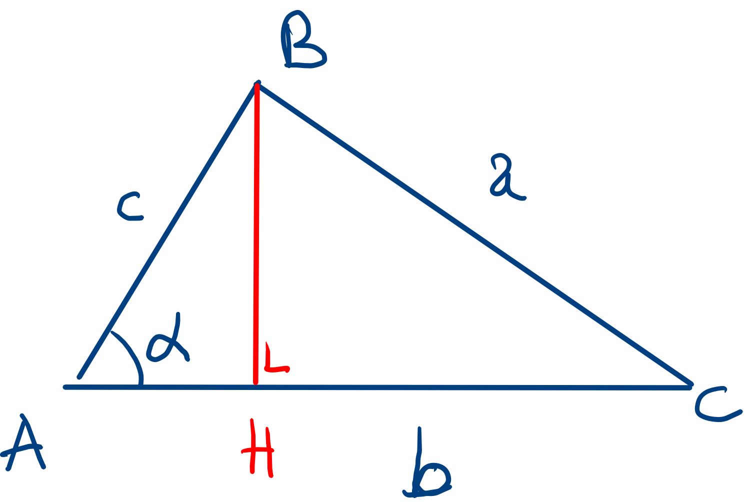
$$\sin(\alpha) = \frac{a}{2r}$$

$$2r = \frac{a}{\sin(\alpha)}$$

$$3 = \frac{6}{2}$$

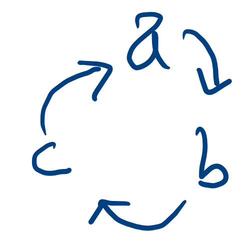
$$\boxed{\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} = 2r}$$

Théorème de l'aire



Aire : $\mathcal{A} = \frac{1}{2} AC \cdot BH$

$$\boxed{\mathcal{A} = \frac{1}{2} b \cdot c \cdot \sin(\alpha)}$$



On calcule BH :

$$\sin(\alpha) = \frac{BH}{c} \Rightarrow BH = c \cdot \sin(\alpha)$$