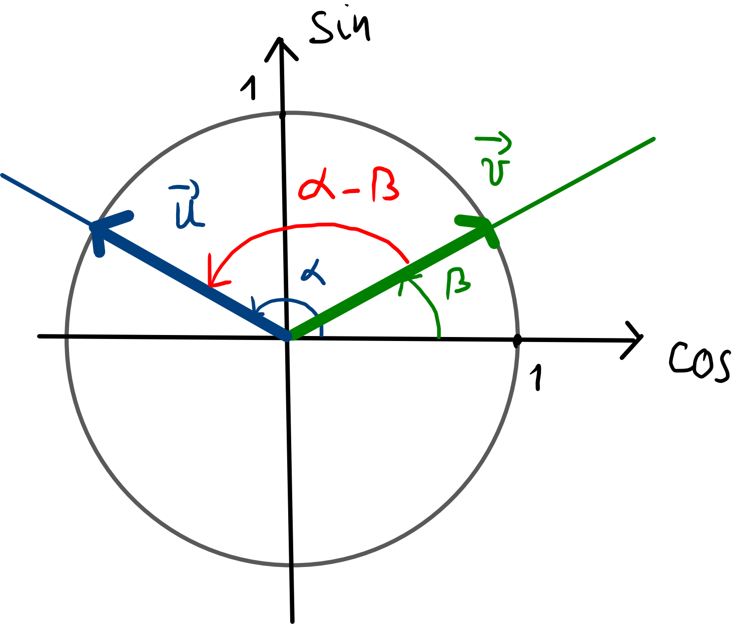
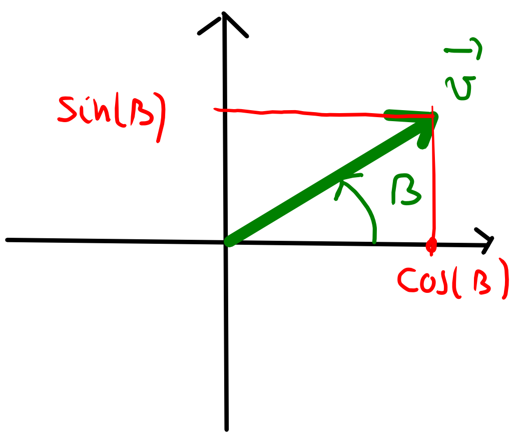
Formules trigonométriques



But : calculer $\cos(\alpha - \beta)$ et $\cos(\alpha + \beta)$

On a : $\vec{u} = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$ et $\vec{v} = \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}$



On calcule le produit scalaire de deux façons:

$$\vec{u} \cdot \vec{v} = \cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta)$$

$$\vec{u} \cdot \vec{v} = \underbrace{\|\vec{u}\|}_{1} \cdot \underbrace{\|\vec{v}\|}_{1} \cos(\alpha - \beta)$$

① $$\boxed{\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)}$$

$$\cos(\alpha + \beta) = \cos(\alpha - (-\beta)) \overset{①}{=} \cos(\alpha) \overbrace{\cos(-\beta)}^{\cos(\beta)} + \sin(\alpha) \overbrace{\sin(-\beta)}^{-\sin(\beta)}$$

$$= \cos(\alpha) \cdot \cos(\beta) - \sin(\alpha)\sin(\beta)$$

② $$\boxed{\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)}$$

Relations entre fonctions trigonométriques de certains arcs

$\cos(-\alpha) = \cos(\alpha)$	$\sin(-\alpha) = -\sin(\alpha)$	$\tan(-\alpha) = -\tan(\alpha)$
$\cos(\pi - \alpha) = -\cos(\alpha)$	$\sin(\pi - \alpha) = \sin(\alpha)$	$\tan(\pi - \alpha) = -\tan(\alpha)$
$\cos(\pi + \alpha) = -\cos(\alpha)$	$\sin(\pi + \alpha) = -\sin(\alpha)$	$\tan(\pi + \alpha) = \tan(\alpha)$
$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$	$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$	$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot(\alpha)$
$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin(\alpha)$	$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos(\alpha)$	$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot(\alpha)$

$$\sin(\alpha - \beta) = \cos\left(\frac{\pi}{2} - (\alpha - \beta)\right) = \cos\left(\left(\frac{\pi}{2} - \alpha\right) + \beta\right)$$

$$\overset{2}{=} \cos\left(\frac{\pi}{2} - \alpha\right)\cos(\beta) - \sin\left(\frac{\pi}{2} - \alpha\right)\cdot\sin(\beta)$$

$$= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

3 $$\boxed{\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)}$$

$$\sin(\alpha + \beta) = \sin(\alpha - (-\beta)) \overset{3}{=} \sin(\alpha)\cos(-\beta) - \cos(\alpha)\sin(-\beta)$$

$$= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

4 $$\boxed{\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)}$$

Applications

But : Calculer la valeur exacte de sin (15°).

α		$\cos(\alpha)$	$\sin(\alpha)$	$\tan(\alpha)$
$0°$	0	1	0	0
$30°$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$90°$	$\frac{\pi}{2}$	0	1	-

$$\begin{aligned}\sin(15°) &= \sin(45° - 30°) \\ &= \sin(45°)\cos(30°) - \cos(45°)\sin(30°) \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\end{aligned}$$

$$\underline{\sin(15°) = \frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$\cos(15°) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

Formule de duplication.

5 $$\boxed{\cos(2\alpha)} \overset{3}{=} \cos(\alpha+\alpha) = \cos(\alpha)\cos(\alpha) - \sin(\alpha)\sin(\alpha) = \boxed{\cos^2(\alpha) - \sin^2(\alpha)}$$

$$= \cos^2(\alpha) - (1 - \cos^2(\alpha)) = \boxed{2\cos^2(\alpha) - 1}$$

6 $$\boxed{\sin(2\alpha) \overset{4}{=} 2\sin(\alpha)\cos(\alpha)}$$