Règles de dérivation

16.01.26

① $(u+v)' = u' + v'$

② $(uv)' = u'v + uv'$

⑧ $$\left(\frac{1}{f(x)}\right)' = \lim_{h\to 0} \frac{\frac{1}{f(x+h)} - \frac{1}{f(x)}}{h} = \lim_{h\to 0} \frac{\frac{f(x)-f(x+h)}{f(x+h)\cdot f(x)}}{h}$$

$$= \underbrace{\lim_{h\to 0} \frac{-\left(f(x+h)-f(x)\right)}{h}} \cdot \frac{1}{f(x+h)\cdot f(x)} =$$

$$= \frac{-f'(x)}{f^2(x)}$$

③ $$\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}$$

Ex : $$\left(\frac{1}{x^2+x}\right)' = \frac{-(2x+1)}{(x^2+x)^2}$$

$$(x^2+x)' = 2x+1$$

$$\left(\frac{1}{\sin(x)}\right)' = \frac{-\cos(x)}{\sin^2(x)}$$

⑨ $$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x)\cdot\frac{1}{g(x)}\right)' \underset{⑦}{=} f'(x)\cdot\frac{1}{g(x)} + f(x)\cdot\left(\frac{1}{g(x)}\right)'$$

$$= \frac{f'(x)}{g(x)} + f(x)\underbrace{\frac{-g'(x)}{g^2(x)}}_{⑧} = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

④ $$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Ex: $$f(x) = \frac{\sin(x)}{x^2};\ f'(x) = \frac{\cos(x)\cdot x^2 - \sin(x)\cdot 2x}{x^4} = \frac{\cancel{x}\left[x\cos(x) - 2\sin(x)\right]}{x^{\cancel{4}\,3}}$$

$u = \sin(x)$; $u' = \cos(x)$

$v = x^2$; $v' = 2x$

$$= \frac{x\cos(x) - 2\sin(x)}{x^3}$$

10 Dérivée d'une fonction composée

$$(\sin(x))' = \cos(x)$$

$$(\sin(x^2))' = 2x\cos(x^2)$$

$$f = g \circ u = g(u) \qquad x \xmapsto{u} u(x) \xmapsto{f} f(u(x))$$

$$f'(a) = \lim_{h \to 0} \frac{(g\circ u)(a+h) - (g \circ u)a}{h}$$

$$= \lim_{h \to 0} \frac{g(u(a+h)) - g(u(a))}{h}$$

Posons $K = u(a+h) - u(a)$; $\lim_{h\to 0} K = 0$

$$= \lim_{h \to 0} \frac{g(u(a)+K) - g(u(a))}{h} \cdot \frac{K}{K}$$

$$= \lim_{h \to 0} \frac{g(u(a)+K) - g(u(a))}{K} \cdot \frac{K}{h}$$

$$= \boxed{\lim_{h \to 0} \frac{g(u(a)+K) - g(u(a))}{K}} \cdot \boxed{\lim_{h \to 0} \frac{u(a+h) - u(a)}{h}}$$

$$= \boxed{g'(u(a))} \cdot \boxed{u'(a)}$$

$$\left[(x^3 - 5x^2)^4\right]' = 4(x^3-5x^2)^3 \cdot \underbrace{(3x^2-10x)}_{\text{dérivée interne}}$$

$g(x) = x^4 \Rightarrow g'(x) = 4x^3$

$u(x) = x^3 - 5x^2 \Rightarrow u' = 3x^2 - 10x$

Ex 2.9.7

Ex 2.9.8

Ex 2.9.10

Ex 2.9.11

2.9.7 Calculer la dérivée de chacune des fonctions suivantes :

a) $f(x) = 47$

b) $f(x) = 3x$

c) $f(x) = x^5$

d) $f(x) = 8x^7$

e) $f(x) = 5x^0$

f) $f(x) = \frac{1}{3}x^3$

g) $f(x) = x^3 + x^2 + x + 1$

h) $f(x) = 7x^4 - 3x + 8$

i) $f(x) = x^2 + 5x - 6$

j) $f(x) = x^3 + 5x^2 - 2x + 4$

k) $f(x) = \frac{2}{3}x^3 - \frac{5}{2}x^2 + 2x + 4$

l) $f(x) = 2x^5 - \frac{7}{6}x^3 + \frac{3}{4}x^2 - x + \sqrt{2}$

$(x)' = 1$

$(3x)' = 3$

$(5x^2)' = 10x$

$\left(-\frac{7}{6}x^3\right)' = \frac{-7}{6} \cdot 3x^2$

$= \frac{-7}{2}x^2$

2.9.8 Calculer la dérivée de chacune des fonctions suivantes :

a) $f(x) = (x+1)(x-3)$

b) $f(x) = x(x^2+5)$

c) $f(x) = (7x^2-4x+3)(5-2x)$

d) $f(x) = (2x-1)(2-2x)(1+x)$

e) $f(x) = \dfrac{4-3x}{2x-1}$

f) $f(x) = \dfrac{x-2}{3-x}$

g) $f(x) = \dfrac{5}{2x^2-1}$

h) $f(x) = \dfrac{x^3-10x^2}{1-x}$

i) $f(x) = \dfrac{8x^2-8x+3}{4x^2-1}$

j) $f(x) = \dfrac{x^3}{x+1}$

k) $f(x) = 1 + \dfrac{1}{x} - \dfrac{2}{x^2}$

l) $f(x) = \dfrac{x^3-4}{3x} + x$

$$(uv)' = u'v + uv'$$

$$(uvw)' = u'vw + uv'w + uvw'$$