22.01.26

2.9.3 Les fonctions suivantes sont-elles dérivables en a ?

a) $f(x) = |x-2|$, $a = 2$

b) $f(x) = x\sqrt{|x|}$, $a = 0$

c) $f(x) = \sqrt{1+\sin(2x)}$, $a = -\frac{\pi}{4}$

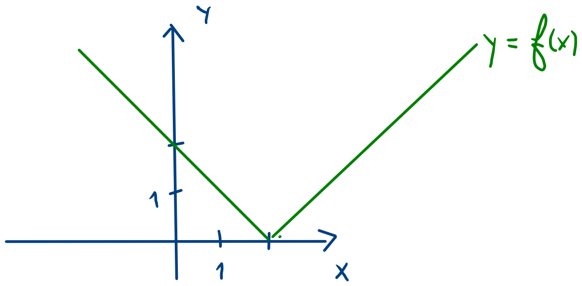
d) $f(x) = |x^2-1| - 2$, $a = -1$

a) $f : \mathbb{R} \longrightarrow \mathbb{R}$

$x \longmapsto |x-2|$

$f(2) = 0$ Non !

$f'(2)$ n'existe pas



b) $f(x) = x\cdot\sqrt{|x|}$, $ED(f) = \mathbb{R}$

$f(0) = 0$, $f'(0)$ existe ?

$$f(x) = \begin{cases} x\sqrt{x} & , \text{ si } x \geq 0 \\ x\sqrt{-x} & \text{ si } x < 0 \end{cases}$$

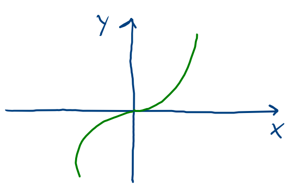
$x \geq 0$: $f'(x) = \left(x\sqrt{x}\right)' = \left(x\cdot x^{\frac{1}{2}}\right)' = \left(x^{\frac{3}{2}}\right)' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

$x < 0$: $f'(x) = \left(x\sqrt{-x}\right)' = \cancel{\left(x\cdot(-x)^{\frac{1}{2}}\right)' = (-x)^{\frac{1}{2}} + x\cdot\frac{1}{2}(-x)^{-\frac{1}{2}}\cdot(-1)}$

$= \cancel{(-x)^{\frac{1}{2}} - \frac{1}{2}x(-x)^{-\frac{1}{2}} =}$

$= \left(-x^{\frac{3}{2}}\right)' = \frac{3}{2}(-x)^{\frac{1}{2}}\cdot(-1) = -\frac{3}{2}\sqrt{-x}$

$f'(0) = 0$



tangente horizontale à la courbe $y = f(x)$ en $x = 0$

$\left[u(v(x))\right]' = u'(v(x))\cdot v'(x)$

$\left[\sin(x^2)\right]' = \cos(x^2)\cdot 2x$

2.9.10 Calculer la dérivée de chacune des fonctions suivantes :

c) $f(x) = (x^2 + 5x + 1)^3$

$$f'(x) = 3(x^2 + 5x + 1)^2 \cdot (\underbrace{2x + 5}_{\text{dérivée interne}})$$

$$[u(v)]' = u'(v) \cdot v'$$

$$x \xrightarrow{v} x^2 + 5x + 1 = y \xrightarrow{u} y^3$$

$$\text{g)}\quad f(x) = \underbrace{\boxed{(2x+5)^3}}_{u}\,\underbrace{\boxed{(3x-1)^4}}_{v}$$

- $u = (2x+5)^3 \quad ; \quad u' = 3(2x+5)^2 \cdot 2 = 6(2x+5)^2$
- $v = (3x-1)^4 \quad ; \quad v' = 4(3x-1)^3 \cdot 3 = 12(3x-1)^3$
- $(uv)' = u'v + uv'$

$$f'(x) = 6(2x+5)^2 \cdot (3x-1)^4 + (2x+5)^3 \cdot 12(3x-1)^3$$

$$= 6(2x+5)^2(3x-1)^3\left[(3x-1) + 2(2x+5)\right]$$

$$f'(x) = 6(2x+5)^2(3x-1)^3(7x+9)$$

k) $f(x) = \dfrac{(1-x)^3}{(1+x)^2} = \dfrac{u}{v}$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$u = (1-x)^3 \; ; \qquad u' = -3(1-x)^2$

$v = (1+x)^2 \; ; \qquad v' = 2(1+x)$

$$f'(x) = \frac{-3(1-x)^2(1+x)^2 - (1-x)^3 \cdot 2(1+x)}{(1+x)^4}$$

$$= \frac{(1-x)^2 \cancel{(1+x)} \left[-3(1+x) - 2(1-x)\right]}{(1+x)^{\cancel{4}\,3}}$$

$$= \frac{(1-x)^2(-3-x-2+2x)}{(1+x)^3} = \frac{(1-x)^2(x-5)}{(1+x)^3}$$

2.9.11 Calculer la dérivée de chacune des fonctions suivantes :

f) $f(x) = \sqrt{(4x^2 - 2x)^3}$ (1°) $= (4x^2 - 2x)^{\frac{3}{2}}$ (2°)

1°) $(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$

$u = (4x^2 - 2x)^3 \;;\; u' = 3(4x^2 - 2x)^2 \cdot (8x - 2)$

$= 6(4x - 1)(4x^2 - 2x)^2$

$$f'(x) = \frac{\overset{3}{\cancel{6}}(4x - 1)(4x^2 - 2x)^2}{\underset{1}{\cancel{2}}\sqrt{(4x^2 - 2x)^3}}$$

2°) $f'(x) = \dfrac{3}{2}(4x^2 - 2x)^{\frac{1}{2}}(8x - 2)$

$$= 3\sqrt{4x^2 - 2x}\,(4x - 1)$$