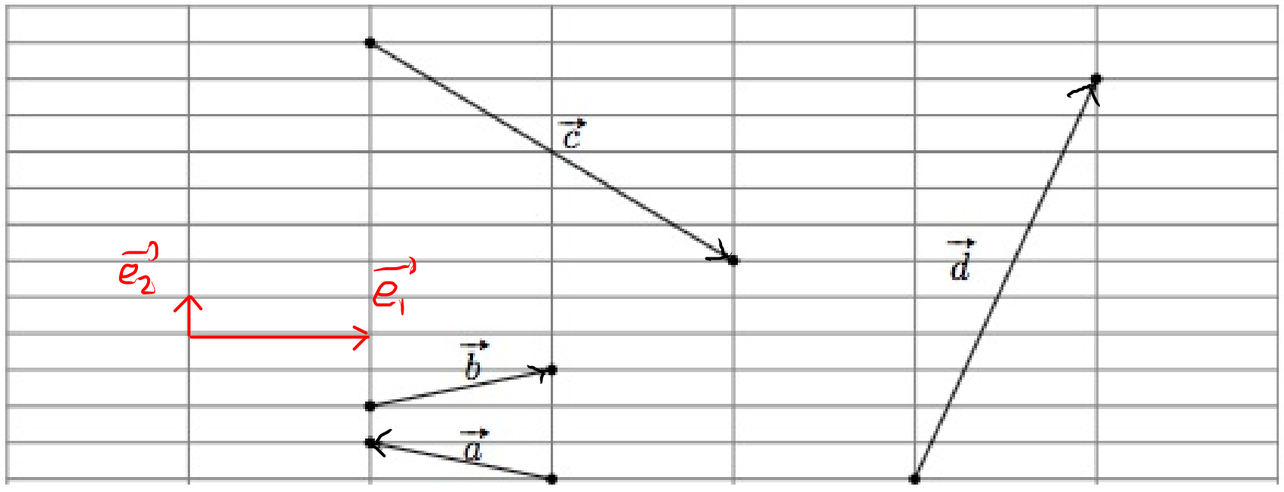
12.09.18

1.2.4 Exprimer les vecteurs $\vec{a}$ et $\vec{b}$ comme combinaison linéaire de $\vec{c}$ et $\vec{d}$ si :



$$\vec{a} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \vec{c} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \qquad \vec{d} = \begin{pmatrix} 1 \\ 11 \end{pmatrix}$$

dans la base $\mathcal{B} = (\vec{e_1}, \vec{e_2})$

$$\begin{cases} \vec{a} = -\vec{e_1} + \vec{e_2} \\ \vec{b} = \vec{e_1} + \vec{e_2} \end{cases} \quad \left| \begin{array}{c|c} \vec{e_1} & \vec{e_2} \\ \cdot 1 & \cdot(-1) \\ \cdot 1 & 1 \end{array} \right.$$

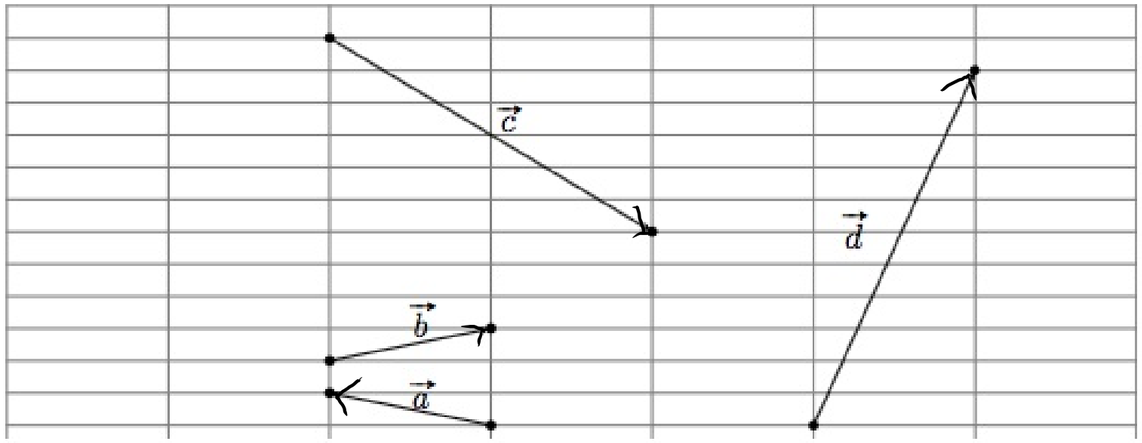
$$\vec{c} = 2\vec{e_1} - 6\vec{e_2}$$
$$\vec{d} = \vec{e_1} + 11\vec{e_2}$$

$$\boxed{\begin{aligned} \vec{e_1} &= -\tfrac{1}{2}\vec{a} + \tfrac{1}{2}\vec{b} \\ \vec{e_2} &= \tfrac{1}{2}\vec{a} + \tfrac{1}{2}\vec{b} \end{aligned}}$$

$$\vec{c} = 2\left(-\tfrac{1}{2}\vec{a} + \tfrac{1}{2}\vec{b}\right) - 6\left(\tfrac{1}{2}\vec{a} + \tfrac{1}{2}\vec{b}\right)$$
$$= -4\vec{a} - 2\vec{b}$$

$$\vec{d} = \left(-\tfrac{1}{2}\vec{a} + \tfrac{1}{2}\vec{b}\right) + 11\left(\tfrac{1}{2}\vec{a} + \tfrac{1}{2}\vec{b}\right)$$

$$= 5\vec{a} + 6\vec{b}$$



$$\begin{cases} \vec{c} = -4\vec{a} - 2\vec{b} \\ \vec{d} = 5\vec{a} + 6\vec{b} \end{cases} \qquad \begin{array}{c|c} \vec{b} & \vec{a} \\ \cdot 3 & \cdot 5 \\ \cdot 1 & \cdot 4 \end{array}$$

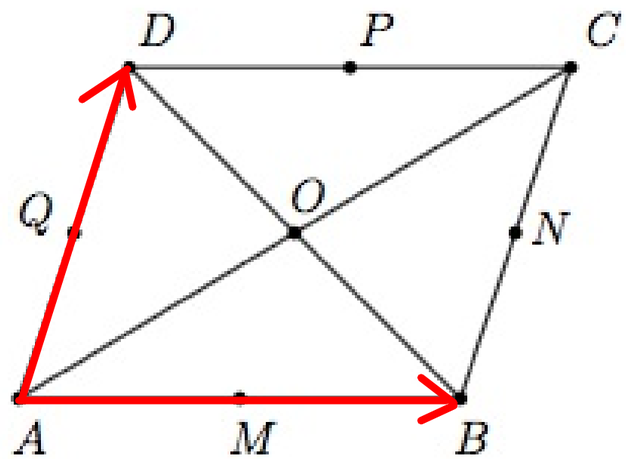
$$3\vec{c} + \vec{d} = -7\vec{a} \quad \Rightarrow \quad \boxed{\vec{a} = -\frac{3}{7}\vec{c} - \frac{1}{7}\vec{d}}$$

$$5\vec{c} + 4\vec{d} = 14\vec{b} \quad \Rightarrow \quad \boxed{\vec{b} = \frac{5}{14}\vec{c} + \frac{2}{7}\vec{d}}$$

Dans la base $\mathcal{B}_1 = (\vec{c}, \vec{d})$:

$$\vec{a} = \begin{pmatrix} -3/7 \\ -1/7 \end{pmatrix}_{\mathcal{B}_1} \qquad \vec{b} = \begin{pmatrix} 5/14 \\ 2/7 \end{pmatrix}_{\mathcal{B}_1}$$

1.2.5 Les points M, N, P et Q sont les milieux des côtés du parallélogramme $ABCD$.



a) Donner, dans la base $\mathfrak{B}_1 = \left(\overrightarrow{AB}; \overrightarrow{AD}\right)$, les composantes des vecteurs $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{AM}$, $\overrightarrow{AQ}$, $\overrightarrow{AN}$, $\overrightarrow{AP}$,$\overrightarrow{AO}$, $\overrightarrow{OB}$, $\overrightarrow{QP}$ et $\overrightarrow{CM}$

b) Mêmes questions, mais relativement à la base $\mathfrak{B}_2 = \left(\overrightarrow{AD}; \overrightarrow{AM}\right)$

$$\overrightarrow{AB} = 1 \cdot \overrightarrow{AB} + 0 \cdot \overrightarrow{AD} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\mathfrak{B}_1}$$

$$\overrightarrow{AD} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; ; \; \overrightarrow{AM} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AN} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$$