

22.05.19

b) $x^5 - 5x^3 + 4x \leq 0$

Factoriser p :

$$\begin{aligned}
 P &= x(x^4 - 5x^2 + 4) \\
 &= x(x^2 - 4)(x^2 - 1) \\
 &= x(x-2)(x+2)(x-1)(x+1)
 \end{aligned}$$

Signe de p :

x	-2	-1	0	1	2	
x	-	-	-	+	+	+
$x-2$	-	-	-	-	-	+
$x+2$	-	0	+	+	+	+
$x-1$	-	-	-	-	0	+
$x+1$	-	-	0	+	+	+
p	-	0	+	0	-	0

$$P = x \left(\underbrace{x^4 - 5x^2 + 4}_{p_1} \right)$$

$$p_1(1) = 0 \Rightarrow x - 1 \not| p_1$$

Horner

The diagram illustrates Horner's method for polynomial division. A horizontal line is divided into four segments by vertical lines. The first segment contains the coefficients of the dividend: 1, 0, -5, 0, 4. A circled '1' is at the start of the first segment, with an arrow pointing to the first vertical line. The second segment starts with a circled '1' under the first vertical line, with an arrow pointing to the second vertical line. The third segment starts with a circled '1' under the second vertical line, with an arrow pointing to the third vertical line. The fourth segment starts with a circled '-4' under the third vertical line, with an arrow pointing to the fourth vertical line. The fifth segment ends with a circled '0' under the fourth vertical line.

$$p_1 = (x-1) \left(x^3 + x^2 - 4x - 4 \right)$$

$$d) (x-2) \cdot (x^2 + 6x - 1) > \underline{(x^2 - 4) \cdot (x^2 + 1)}$$

$$(x-2)(x^2 + 6x - 1) - \underline{(x-2)(x+2)(x^2 + 1)} > 0$$

$$(x-2) \left[(x^2 + 6x - 1) - (x+2)(x^2 + 1) \right] > 0$$

$$(x-2) \left(x^2 + 6x - 1 - (x^3 + x + 2x^2 + 2) \right) > 0$$

$$(x-2) (-x^3 - x^2 + 5x - 3) > 0$$

$$-(x-2) (x^3 + x^2 - 5x + 3) > 0$$

$$(x-2) (x^3 + x^2 - 5x + 3) < 0$$

P

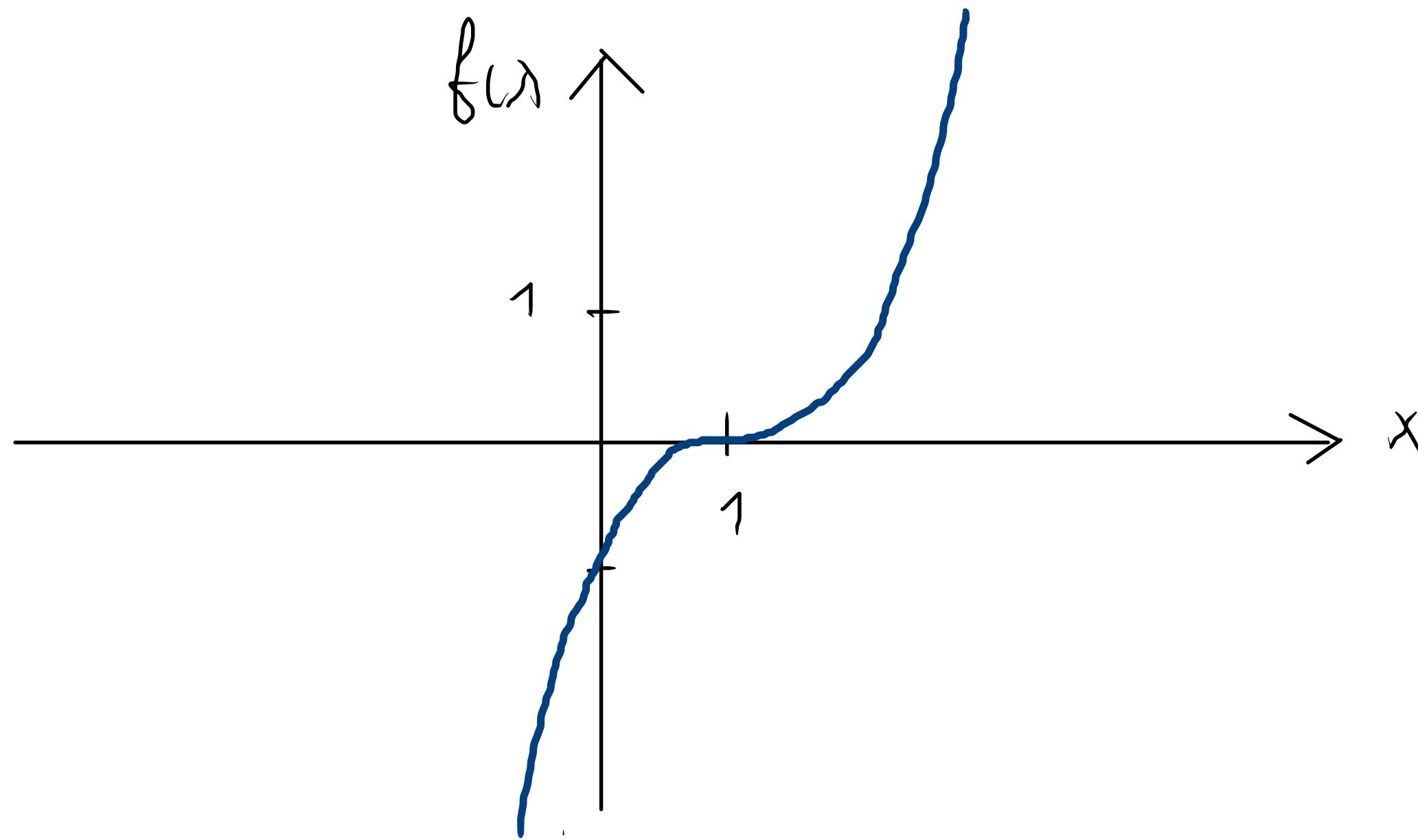
3.4.28 Établir le tableau des signes des polynômes suivants, puis esquisser les graphes des fonctions correspondantes

a) $f(x) = x^4 - x^3 - 11x^2 + 9x + 18$

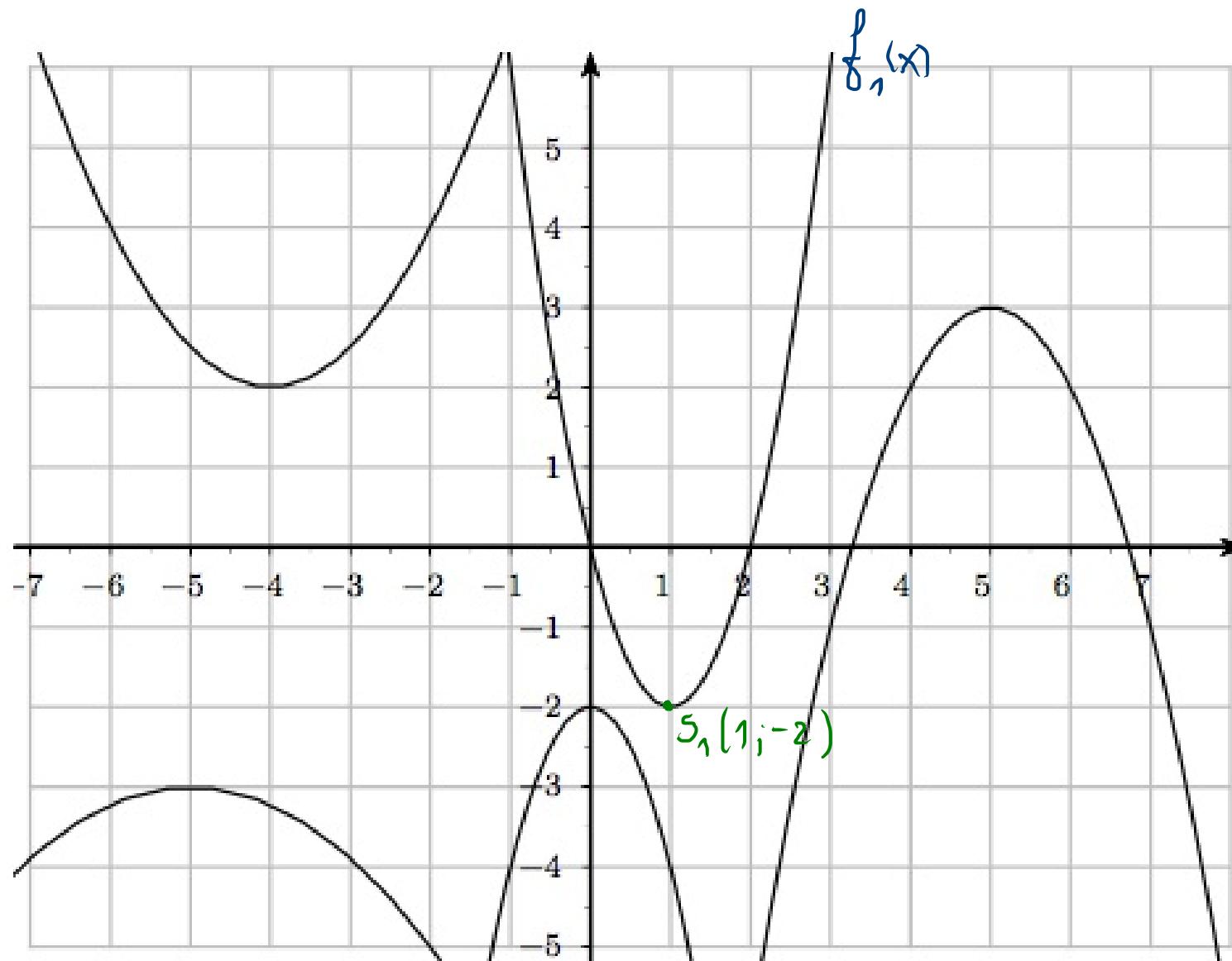
b) $f(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1 = (x - 1)^5$

b)

x		1	
$f(x)$	-	∅	+



3.4.22



$$f_1(x) = ax^2 + bx + c = a(x-\alpha)(x-\beta)$$

où α et β sont les zéros de f_1

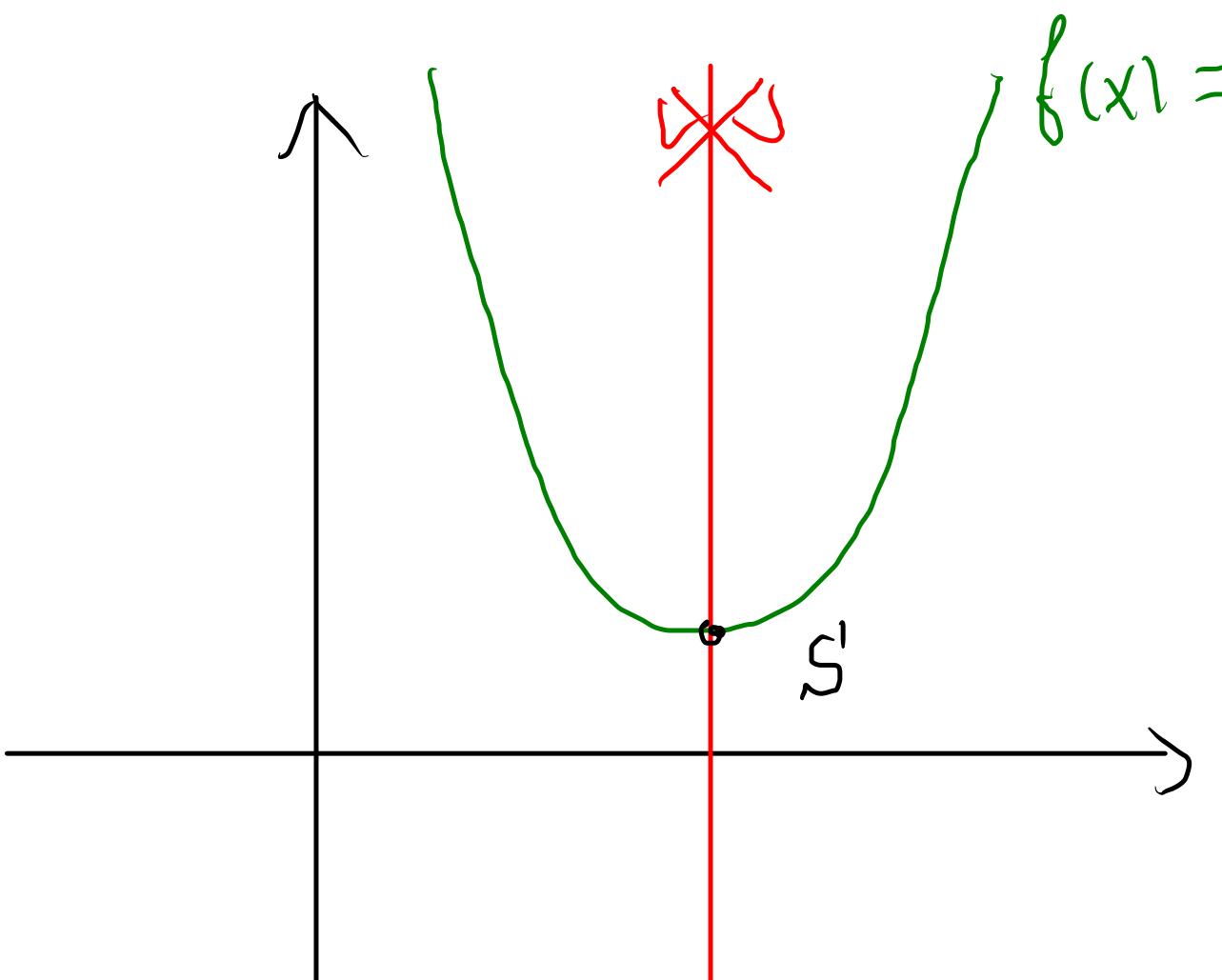
$$f_1(x) \neq a(x-0)(x-2) = ax(x-2)$$

Comme $f_1(1) = -2$, $a \cdot 1(1-2) = -2$

donc $-a = -2$, $a = 2$

Ainsi $f_1(x) = 2x(x-2) = 2x^2 - 4x$

La parabole (suite)

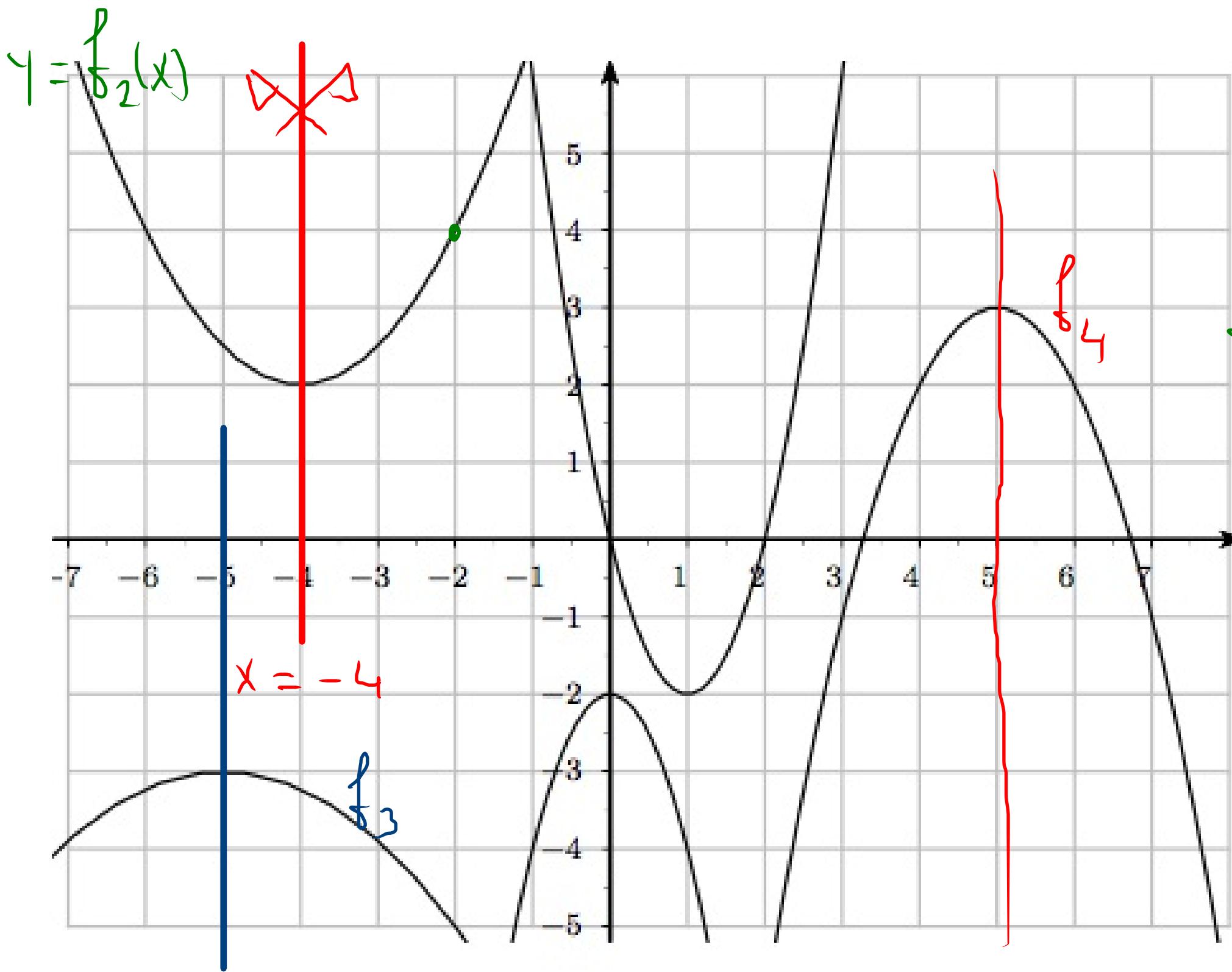


$$x = -\frac{b}{2a}$$

$$\begin{aligned}f(x) &= a \left(x^2 + \frac{b}{a}x + \frac{c}{a} \right) \\&= a \left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a} \right)^2 - \left(\frac{b}{2a} \right)^2 + \frac{c}{a} \right) \\&= a \left(x + \frac{b}{2a} \right)^2 - a \frac{b^2 - 4ac}{4a^2} \\&= a \left(x + \frac{b}{2a} \right)^2 - \frac{b^2 - 4ac}{4a}\end{aligned}$$

$$S \left(-\frac{b}{2a}, \frac{-b^2 + 4ac}{4a} \right)$$

$S \left(-\frac{b}{2a}, \frac{-b^2 + 4ac}{4a} \right)$ est le sommet de la parabole



$$f_2(x) = \frac{1}{2}(x+4)^2 + 2$$

$$= \frac{1}{2}x^2 + 4x + 10$$

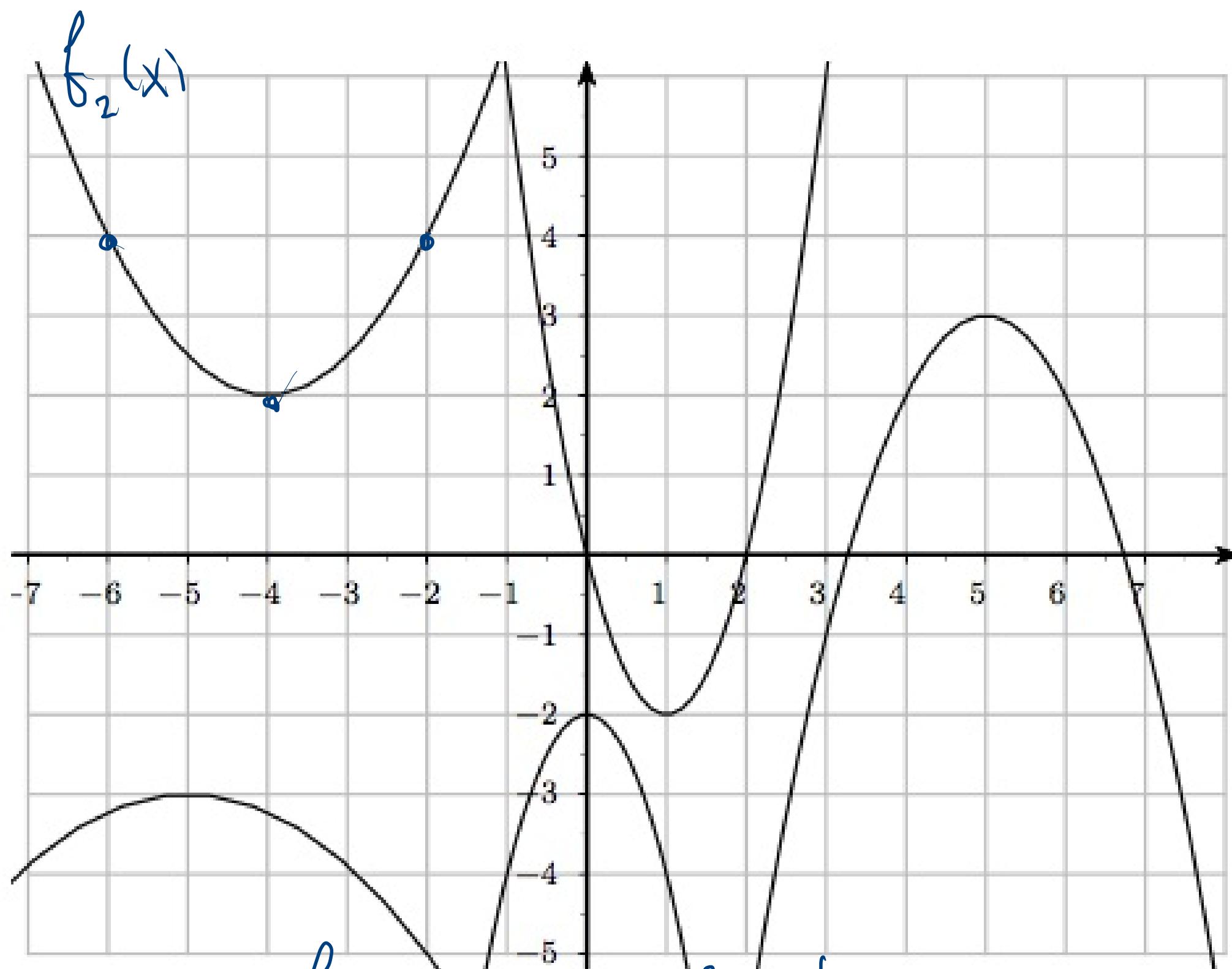
$$f_3(x) = 2(x+6)^2 - 3$$

$$f_4(x) = 2(x-5)^2 + 3$$

$$f_2(x) = 2(x+4)^2 + 2$$

$$f_2(-2) = 4, \Rightarrow 2(-2+4)^2 + 2 = 4$$

$$4a = 2 \Rightarrow a = \frac{1}{2}$$



$$f_2(x) = ax^2 + bx + c$$

$$\begin{cases} f_2(-6) = 4 \\ f_2(-4) = 2 \\ f_2(-2) = 4 \end{cases}$$

$$\Rightarrow \begin{cases} 36a - 6b + c = 4 \\ 16a - 4b + c = 2 \\ 4a - 2b + c = 4 \end{cases}$$