a) $E = \{x \in \mathbb{Z} \mid -2 < x \leq +2\}$

- $E \underset{\neq}{\subset}]-2;2]$ donc E minoré, majoré et borné

- $\min(E)$ ~~n'existe pas~~ -1 dans $\mathbb{Z}$

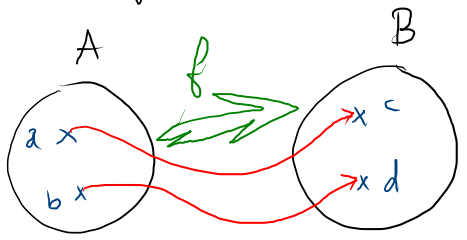
  $\max(E) = 2$

  $\inf(E) = -2$ dans $\mathbb{R}$

  $\sup(E) = 2$

Fonction injective, surjective

① Fonction injective

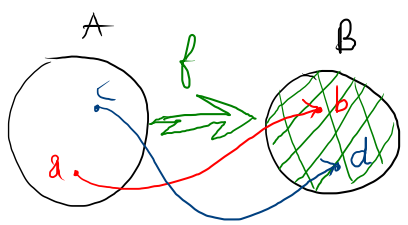


$$\boxed{a, b \in A \ , \ f(a) = f(b) \iff a = b}$$

Contre-exemple : $f(x) = 5$
$f(x) = x^2$

Exemple : $f(x) = 2x$
$f(x) = x^3$

② Fonction surjective



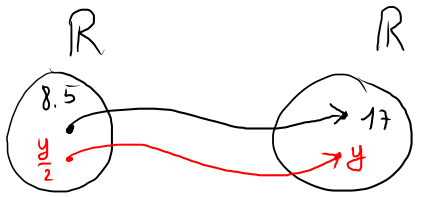
$f(A) = B$

$\text{Image}(f) = B$

$$\forall b \in B \ , \ \exists a \in A \text{ tel que } f(a) = b$$

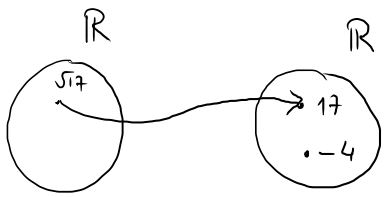
Exemple

$f : \mathbb{R} \longrightarrow \mathbb{R}$
$x \longmapsto 2x$



Contre-exemple

$f : \mathbb{R} \longrightarrow \mathbb{R}$
$x \longmapsto x^2$



Une fonction injective et surjective est bijective

bijective

$$2^x : \mathbb{R} \longrightarrow \mathbb{R}_+^*$$

$$\log_2(x) \quad \mathbb{R} \longleftarrow \mathbb{R}_+^*$$

$$10^x : \mathbb{R} \longrightarrow \mathbb{R}_+^*$$

$$x \longmapsto 10^x$$

$$\boxed{\begin{array}{l} 10^{1,89} = 77.624711662869173 \\ \log(77.624711662869173) = 1,89 \end{array}}$$

$$\log(x) : \mathbb{R}_+^* \longrightarrow \mathbb{R}$$

$$x \longmapsto \log(x)$$

2.3.1 Déterminer les applications injectives, surjectives ou bijectives.

a) $f_1 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ x & \mapsto & 2x+1 \end{array}$

b) $f_2 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ x & \mapsto & x^2 \end{array}$

c) $f_3 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ x & \mapsto & x-3 \end{array}$

d) $f_4 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ x & \mapsto & x^3 \end{array}$

e) $f_5 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ x & \mapsto & x \end{array}$

f) $f_6 : \begin{array}{ccc} \mathbb{N} & \to & \mathbb{N} \\ x & \mapsto & x+2 \end{array}$

g) $f_7 : \begin{array}{ccc} \mathbb{N} & \to & \mathbb{N} \\ x & \mapsto & x^2 \end{array}$

h) $f_8 : \begin{array}{ccc} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & x^2 \end{array}$

i) $f_9 : \begin{array}{ccc} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & 2x+1 \end{array}$

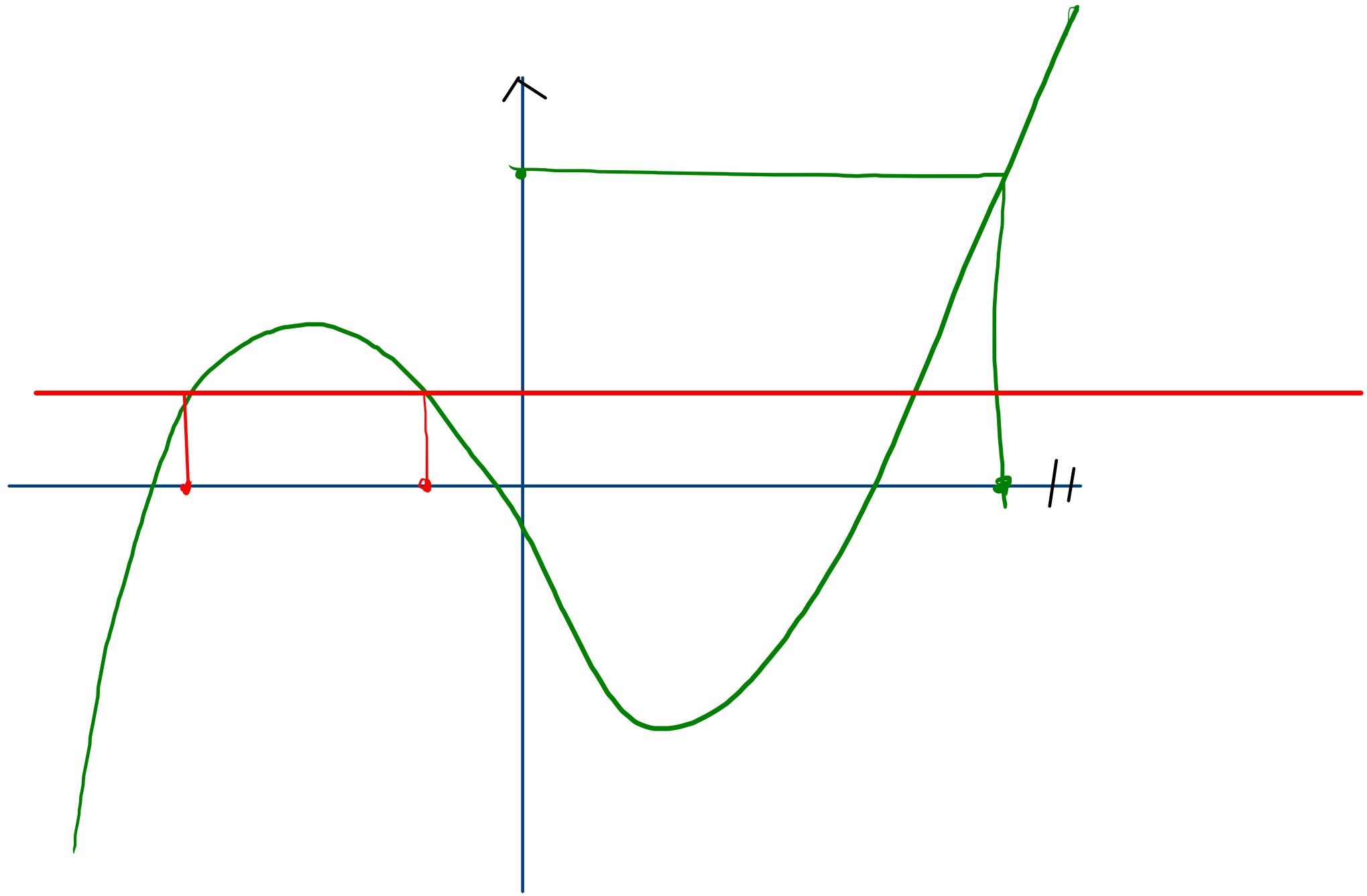
a) f_1 est injective. En effet, soit $a, b \in \mathbb{Z}$ tel que $f_1(a) = f_1(b)$.

$$\begin{aligned} 2a+1 &= 2b+1 \quad \Big| -1 \\ 2a &= 2b \quad \Big| \div 2 \\ a &= b \end{aligned}$$

donc $f_1(a) = f_1(b) \iff a = b$

f_1 n'est pas surjective. En effet, si $a \in \mathbb{Z}$ est tel que $f(a) = 0$, alors $2a+1 = 0 \iff a = -\frac{1}{2}$

$a \notin \mathbb{Z}$.



b) $f_2 : \mathbb{Z} \to \mathbb{Z}$
$x \mapsto x^2$

- $f_2(1) = f_2(-1) = 1$ pas injective
- $-3, 2, ---$ pas surjectif

c) $f_3 : \mathbb{Z} \to \mathbb{Z}$
$x \mapsto x - 3$

- injectif ✓
- surjectif $y \in \mathbb{Z}$, $f(y+3) = y$

bijectif

$$f(x) = x^2$$

$$a \longmapsto a^2$$
$$b \longmapsto b^2$$

$$a^2 = b^2$$
$$a^2 - b^2 = 0$$
$$(a-b)(a+b) = 0$$

$$a = b \qquad a = -b$$

g) $$\begin{array}{rcl} f_7 : \mathbb{R}-\{1\} & \to & \mathbb{R}-\{2\} \\ x & \mapsto & \dfrac{2x+1}{x-1} \end{array}$$

$$\frac{2x+1}{x-1} = 4$$

$$2x+1 = 4x-4$$

$$-2x = -5$$

$$x = \frac{5}{2}$$

$y \in \mathbb{R}-\{2\}$

$$\frac{2x+1}{x-1} = y \qquad \cdot (x-1) \neq 0$$

$$2x+1 = (x-1)y$$

$$2x+1 = xy - y$$

$$2x - xy = -1-y$$

$$x(2-y) = -1-y$$

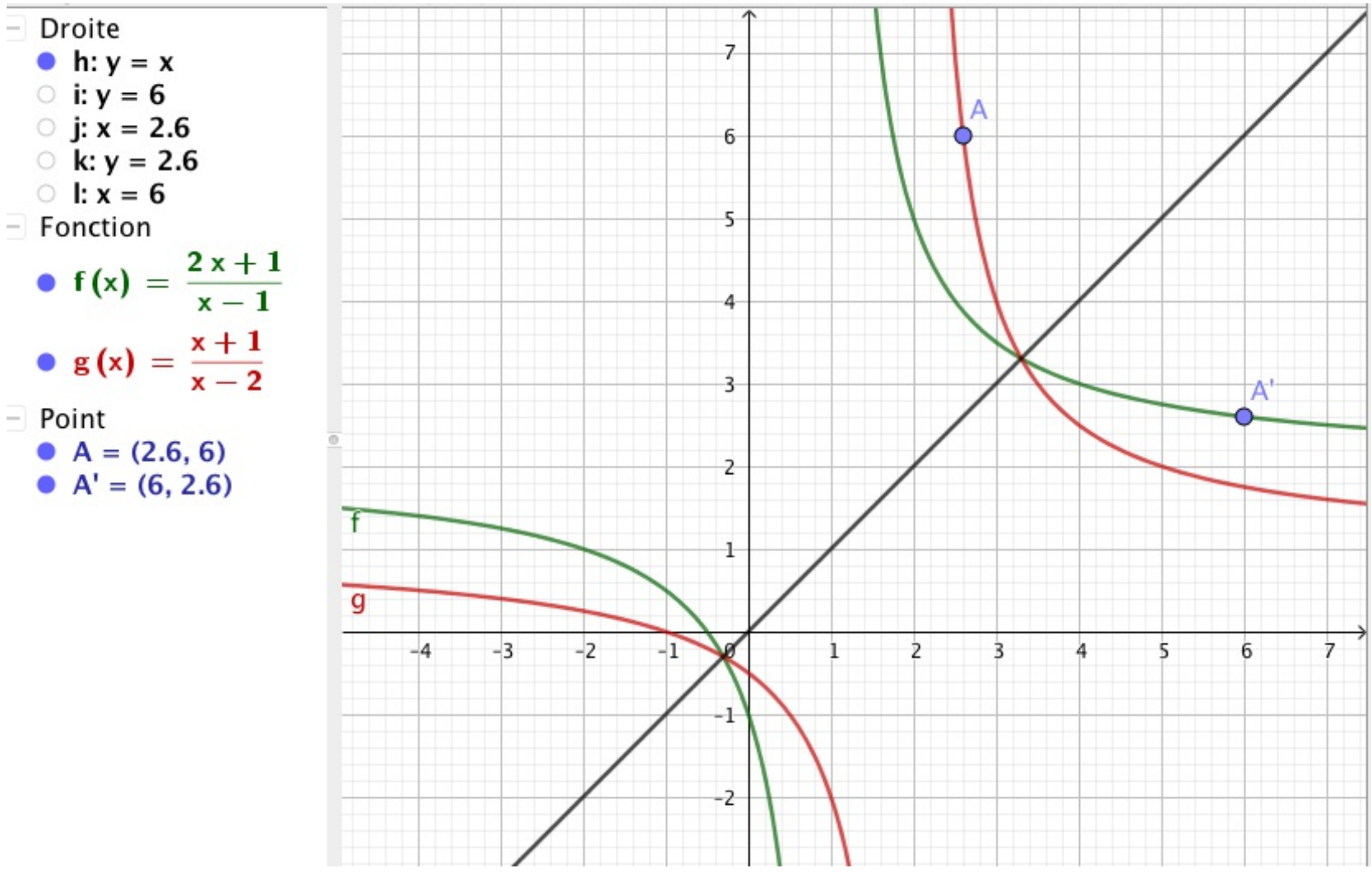
$$x = \frac{-1-y}{2-y}$$

$$x = \frac{y+1}{y-2}$$

$y \neq 2$

$$f_7^{r} : \mathbb{R}-\{2\} \longrightarrow \mathbb{R}-\{1\}$$

$$y \longmapsto \frac{y+1}{y-2}$$


Droite
h: y = x
i: y = 6
j: x = 2.6
k: y = 2.6
l: x = 6
Fonction
f(x) = (2x + 1)/(x − 1)
g(x) = (x + 1)/(x − 2)
Point
A = (2.6, 6)
A' = (6, 2.6)
A
A'
f
g