

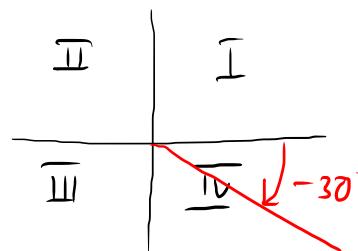
05.09.19

Exercice 22 Calculer le module et un argument des complexes suivants, puis les écrire sous formes trigonométrique et exponentielle :

$$z_1 = \frac{\sqrt{6} - i\sqrt{2}}{2(1+i)}$$

$$z_2 = \frac{5(-1+i)}{\sqrt{3}+i}$$

$$z_2 = \sqrt{6} - i\sqrt{2} =$$



$$|z_2| = \sqrt{8}$$

$$\begin{cases} \cos(\theta_1) = \frac{\sqrt{6}}{\sqrt{8}} = \frac{\sqrt{6}\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}}{2} \\ \sin(\theta_1) = \frac{-\sqrt{2}}{\sqrt{8}} = \frac{-\sqrt{2}}{2\sqrt{2}} = -\frac{1}{2} \end{cases} \quad \theta_1 = 330^\circ$$

$$\underline{z_2 = [\sqrt{8}; \frac{11\pi}{6}]}$$

$$z_3 = 2(1+i)$$

$$|z_3| = 2\sqrt{2}$$

$$\begin{cases} \cos(\theta_2) = \frac{1}{\sqrt{2}} \\ \sin(\theta_2) = \frac{1}{\sqrt{2}} \end{cases} \quad \theta_2 = \frac{\pi}{4}$$

$$\underline{z_3 = [\sqrt{8}; \frac{\pi}{4}]}$$

$$z_2 \div z_3 = [1; \frac{11\pi}{6} - \frac{\pi}{4}] = [1; \frac{19\pi}{12}]$$

$$\boxed{z_1 = \cos\left(\frac{19\pi}{12}\right) + i \sin\left(\frac{19\pi}{12}\right)}$$

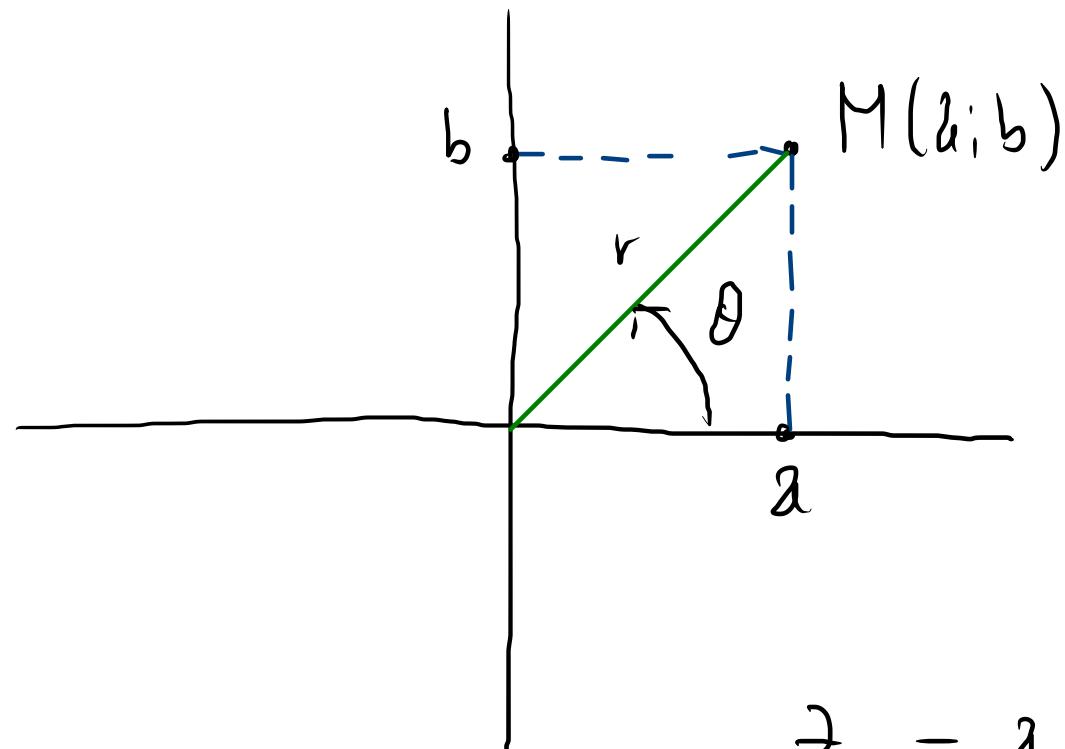
$$z_1 \approx 0.258819045102521 + i(-0.965925826289068)$$

$$285^\circ$$

$$270^\circ + 15^\circ$$

$$r = \sqrt{a^2 + b^2}$$

$$\cos(\theta) = \frac{a}{r}$$



$$\sin(\theta) = \frac{b}{r}$$

$$z_M = a + bi$$

notation

$$z = r \left(\underbrace{\cos(\theta) + i \sin(\theta)}_{\text{ }} \right) = [r; \theta]$$

$$= r \cdot e^{i\theta}$$

On note $[r, \theta]$ le nombre complexe

$$z = r \cos(\theta) + r \sin(\theta) i, \text{ avec } r, \theta \in \mathbb{R}, r > 0$$

Proposition

Soit $z, z' \in \mathbb{C}$. On peut écrire $z = [r, \theta]$ et
 $z' = [r', \theta']$.

$$\textcircled{1} \quad z \cdot z' = [rr'; \theta + \theta']$$

$$\textcircled{2} \quad z \div z' = \left[\frac{r}{r'} ; \theta - \theta' \right]$$

$$\textcircled{3} \quad z^n = [r^n, n\theta], \quad n \in \mathbb{N}^*$$

Démonstration

$$\begin{aligned}
 ① z \cdot z' &= (r \cos \theta + r \sin \theta i)(r' \cos \theta' + r' \sin \theta' i) \\
 &= rr' \left[(\cos \theta \cos \theta' - \sin \theta \sin \theta') + (\sin \theta \cos \theta' + \sin \theta' \cos \theta) i \right] \\
 &= rr' \left(\underline{\cos(\theta + \theta')} + \underline{\sin(\theta + \theta')} i \right) \\
 &\text{Table} \\
 &= [rr'; \theta + \theta']
 \end{aligned}$$

② En exercice

$$③ [r, \theta]^n = [r^n, n\theta]$$

Démontrons ce résultat par récurrence sur n .

a) Démontrons que le résultat est vrai pour $n = 1$.

$$[r, \theta]^1 = [r^1, 1 \cdot \theta] = [r, \theta]$$

b) Supposons le résultat vrai pour n et démontre-le pour $n+1$.

$$\begin{aligned}
 [r, \theta]^{n+1} &= [r, \theta] \cdot [r, \theta]^n = [r, \theta] \cdot [r^n, n\theta] \\
 &\stackrel{\substack{\text{par hypo.} \\ \text{de récurrence}}}{=} [r \cdot r^n; \theta + n\theta] = [r^{n+1}; (n+1)\theta] \\
 &\quad \text{cqfd}
 \end{aligned}$$

1.2.6 ; 1.2.8 ; 1.2.9

1.2.8 Déterminer :

a) la forme trigonométrique des nombres complexes z tel que $z^4 = \underbrace{1+i}_{\omega}$

$$\omega = 1+i \quad , \quad \omega = \sqrt{2}$$

$$= [\sqrt{2}; \frac{\pi}{4}]$$

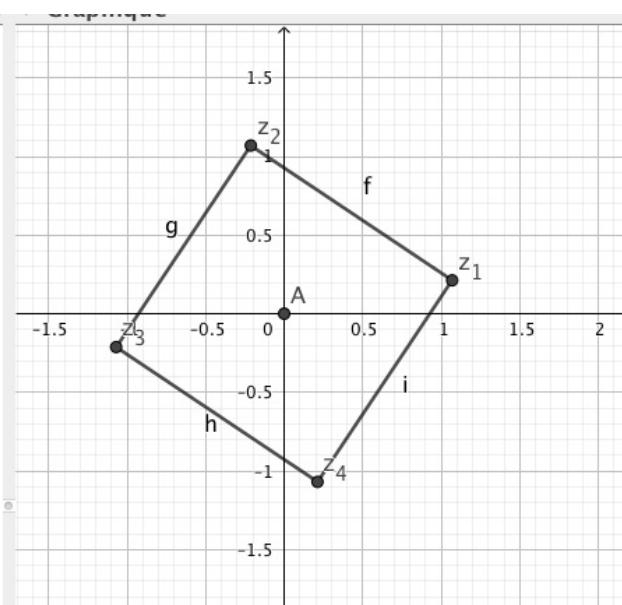
Soit $z = [r, \theta]$ tel que $z^4 = [\sqrt{2}, \frac{\pi}{4}]$

Donc $[r^4, 4\theta] = [\sqrt{2}, \frac{\pi}{4}]$

$$\begin{cases} r^4 = \sqrt{2} \\ 4\theta = \frac{\pi}{4} + 2k\pi \end{cases} \Rightarrow \begin{cases} r = \sqrt[4]{\sqrt{2}} = \sqrt[8]{2} \\ \theta = \frac{\pi}{16} + k \cdot \frac{\pi}{2}, \quad k \in \mathbb{Z} \end{cases}$$

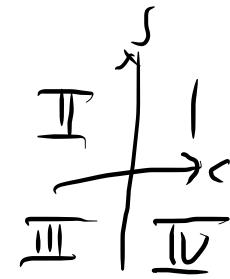
$$[\sqrt[8]{2}; \frac{\pi}{16}], [\sqrt[8]{2}; \frac{9\pi}{16}], [\sqrt[8]{2}; \frac{17\pi}{16}], [\sqrt[8]{2}; \frac{25\pi}{16}]$$

Nombre	
○	$a = 1.09051$
Nombre complexe	
●	$z_1 = 1.06955 + 0.21275i$
●	$z_2 = -0.21275 + 1.06955i$
●	$z_3 = -1.06955 - 0.21275i$
●	$z_4 = 0.21275 - 1.06955i$
Point	
●	$A = (0, 0)$
Segment	
●	$f = 1.54221$
●	$g = 1.54221$
●	$h = 1.54221$
●	$i = 1.54221$



1.2.5 Calculer $z_1 z_2$ et z_1/z_2 en utilisant la forme trigonométrique:

a) $z_1 = -1 + i, \quad z_2 = 1 + i$



$$\textcircled{1} \quad z_1 = -1 + i = [\sqrt{2}; \frac{3\pi}{4}]$$

$$\begin{cases} \cos(\theta) = \frac{-1}{\sqrt{2}} \\ \sin(\theta) = \frac{1}{\sqrt{2}} \end{cases} \Rightarrow \theta = 135^\circ$$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$z_2 = [\sqrt{2}, \frac{\pi}{4}]$$

$$z_1 \cdot z_2 = [2; \pi] = -2$$

$$\frac{z_1}{z_2} = [1; \frac{\pi}{2}] = i$$

$$\textcircled{2} \quad \frac{-1+i}{1+i} \cdot \frac{1-i}{1-i} = \frac{-1+i+i+1}{2} = \frac{2i}{2} = i = [1; \frac{\pi}{2}]$$