

2.8.26 Déterminer les nombres réels a et b pour lesquels les courbes $y = x^3 + ax^2 + bx$ et $y = x^2 - 6x$ sont tangentes en un point d'abscisse 4.

$$(C_1): y = x^3 + ax^2 + bx \Rightarrow y = f(x), f'(x) = 3x^2 + 2ax + b$$

$$(C_2): y = x^2 - 6x \Rightarrow y = g(x), g'(x) = 2x - 6$$

$$C_1 = \{ (x, y) \mid y = x^3 + ax^2 + bx \}$$

Les pentes des tangentes au Point $T(4; -8)$:

$$\left. \begin{array}{l} f'(4) = 48 + 8a + b \\ g'(4) = 2 \end{array} \right\} \quad \begin{array}{l} 48 + 8a + b = 2 \\ \textcircled{1} \end{array}$$

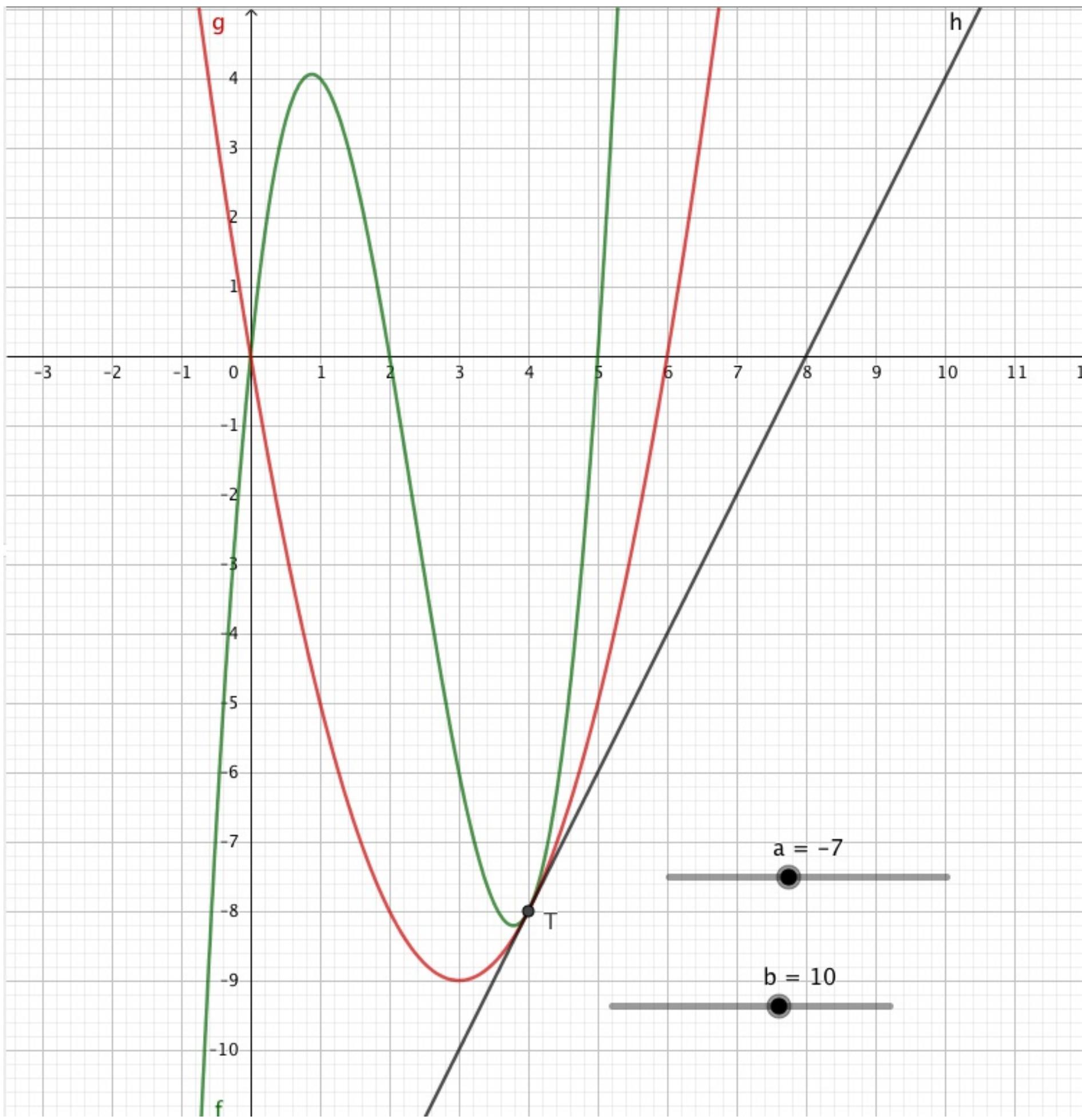
$T \in C_1$ et $T \in C_2$

$$\textcircled{2} \quad 64 + 16a + 4b = -8$$

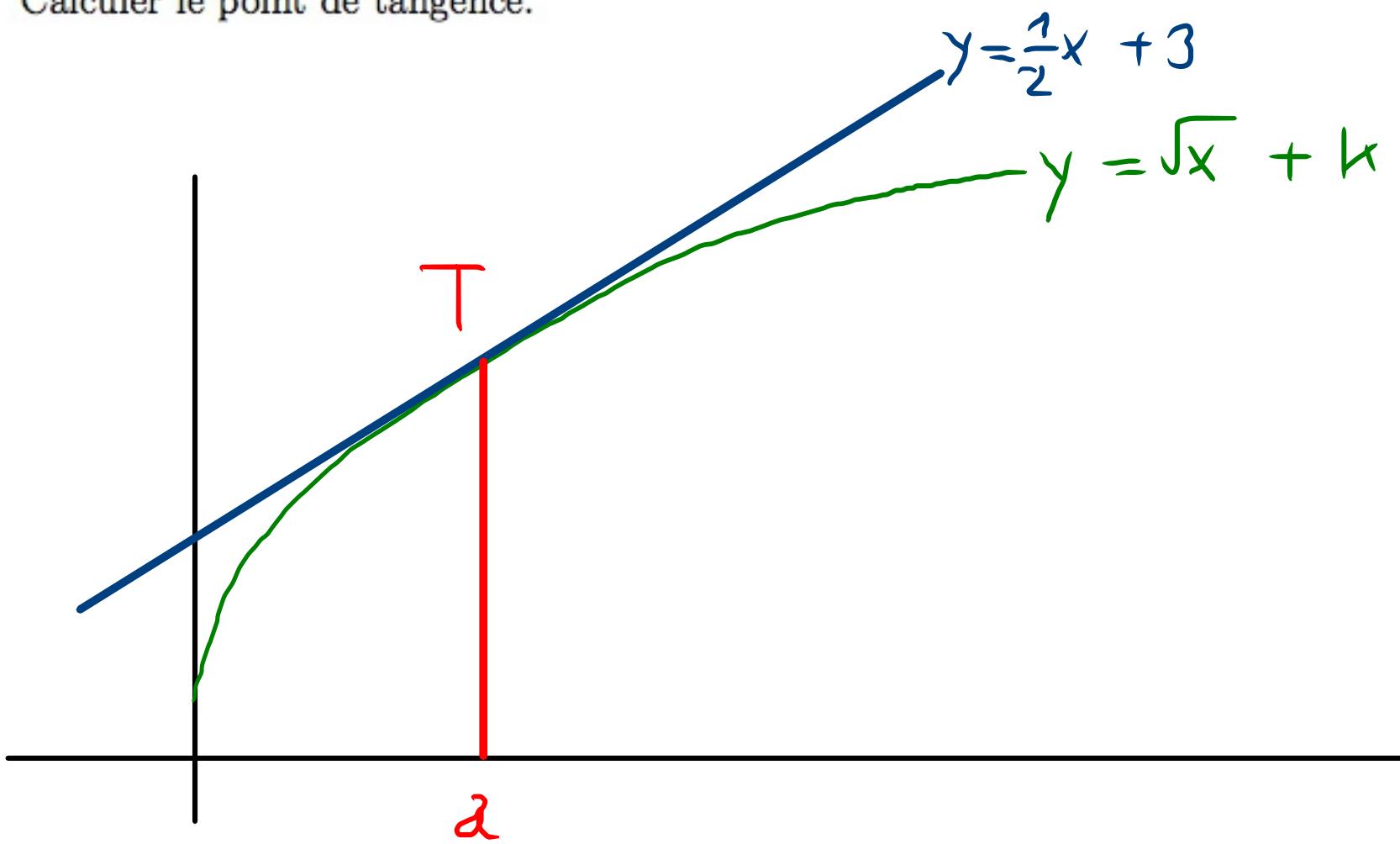
On résout le système:

$$\left. \begin{array}{l} \textcircled{1} \quad \left\{ \begin{array}{l} 8a + b = -46 \\ 16a + 4b = -72 \end{array} \right. \\ \textcircled{2} \quad \left. \begin{array}{l} \cdot 4 \\ \cdot (-1) \end{array} \right| \begin{array}{c} b \\ a \end{array} \end{array} \right| \begin{array}{l} \cdot 4 \\ \cdot (-1) \end{array}$$

$$\left. \begin{array}{l} 16a = -112 \\ -2b = -20 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = -7 \\ b = 10 \end{array} \right\}$$



2.8.27 Déterminer $k \in \mathbb{R}$ pour que les courbes $y = \sqrt{x} + k$ et $y = \frac{x}{2} + 3$ soient tangentes.
 Calculer le point de tangence.



$$f(x) = \sqrt{x} + k$$

$$g(x) = \frac{1}{2}x + 3$$

$$f'(x) = \frac{1}{2}$$