

06.09.19

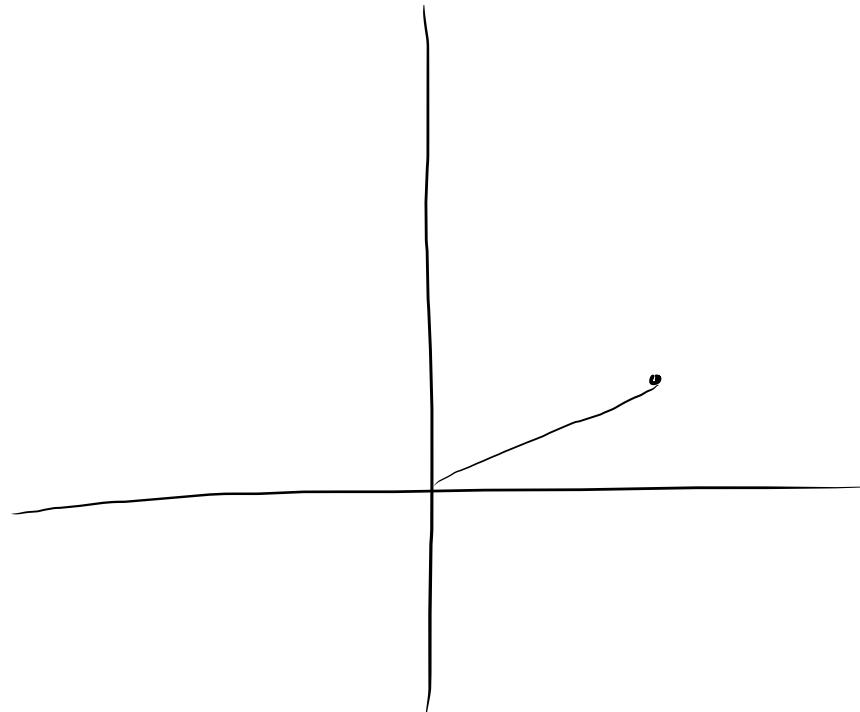
Nombre complexe 1

$$1) 87i$$

$$2) \frac{34}{25}$$

$$3) z = [2; \frac{7\pi}{6}]$$

$$4) 512 - \sqrt{3} \cdot 512i = 512(1 - \sqrt{3}i)$$
$$= [1024, \frac{5\pi}{3}]$$



1.2.8 Déterminer :

- a) la forme trigonométrique des nombres complexes z tel que $z^4 = 1 + i$,
- b) la forme algébrique des nombres complexes z tel que $z^4 = 24i - 7$.

$$b) \quad w = -7 + 24i \quad \simeq [25; 106,26^\circ]$$

106.260204708311957

$$z = a + bi = [r, \theta]$$

$$z^4 = [r^4, 4\theta]$$

$$\begin{cases} r^4 = 25 \\ 4\theta \simeq 106,26^\circ + k360^\circ \end{cases} \Rightarrow \begin{cases} r = \sqrt[4]{25} \\ \theta = \underbrace{26.565051177077989}_{\alpha} + k90^\circ \end{cases}$$

$$z_1 = \sqrt[4]{25} \left(\cos(\alpha) + i \sin(\alpha) \right) = 2 + i$$

$$z_2 = -1 + 2i$$

$$z_3 = -2 - i$$

$$z_4 = 1 - 2i$$

1.2.9 Déterminer sous forme trigonométrique et représenter dans le plan complexe :

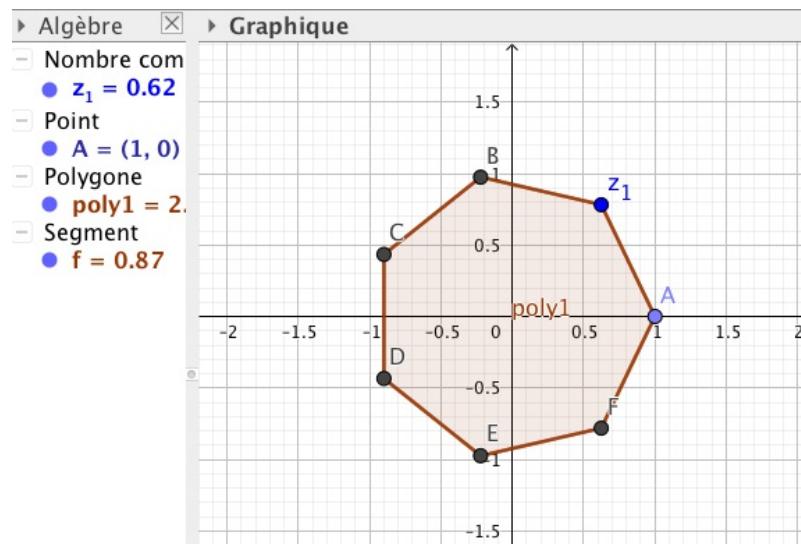
- les racines septièmes de l'unité,
- les solutions de l'équation $z^5 = -32$.

$$\text{a) } z^7 = 1$$

$$z^7 = [1; 0]$$

Posons $z = [r, \theta]$

$$\text{Donc } [r^7, 7\theta] = [1, 0]$$



$$\begin{cases} r^7 = 1 \\ 7\theta = 2k\pi \end{cases} \Rightarrow \begin{cases} r = 1 \\ \theta = \frac{k}{7} \cdot 2\pi \end{cases} \quad k \in \mathbb{Z}$$

$$z_1 = [1, 0], z_2 = [1, \frac{2\pi}{7}], z_3 = [1, \frac{4\pi}{7}], z_4 = [1, \frac{6\pi}{7}]$$

$$z_5 = [1, \frac{8\pi}{7}], z_6 = [1, \frac{10\pi}{7}], z_7 = [1, \frac{12\pi}{7}]$$

$$z_1 = \cos(2\pi/7) + \sin(2\pi/7)i$$

1.2.10 Déterminer sous forme algébrique les racines carrées complexes des nombres ci-dessous :

a) 1

d) -9

b) i

e) $3 + 4i$

c) $-i$

f) $-5 + 12i$

a) $z^2 = 1$

$z = \pm 1$

$S = \{-1, 1\}$

b) $z^2 = i$ $|z|^2 = 1$

$z = a + bi$, $z^2 = a^2 - b^2 + 2abi$

D'où le système d'équation

$$\begin{array}{l} \text{réelle} \\ \text{imaginai} \\ \text{modul} \end{array} \left\{ \begin{array}{l} a^2 - b^2 = 0 \\ 2ab = 1 \\ a^2 + b^2 = 1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} a^2 - b^2 = 0 \\ a^2 + b^2 = 1 \\ 2ab = 1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} 2a^2 = 1 \\ 2b^2 = 1 \\ 2ab = 1 \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} a^2 = \frac{1}{2} \\ b^2 = \frac{1}{2} \\ ab = \frac{1}{2} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} a = \pm \frac{1}{\sqrt{2}} \\ b = \pm \frac{1}{\sqrt{2}} \\ ab = \frac{1}{2} \end{array} \right.$$

$z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i ; \quad z_2 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$