07.11.19

4.1.16 Calculer :

a) $8^{\frac{2}{3}} + 16^{\frac{1}{2}} + 27^{\frac{2}{3}} + 81^{\frac{1}{4}} - 125^{\frac{1}{3}} - 1'000^{\frac{2}{3}}$

b) $(3 \cdot 32^{\frac{1}{3}} + 3 \cdot 108^{\frac{1}{3}} - 256 \cdot 2^{\frac{2}{3}}) \cdot 2^{\frac{1}{3}}$

c) $(3 \cdot 2^{0,25} + 2 \cdot 32^{0,25} - 8^{0,75}) \cdot 8^{0,25}$

d) $\dfrac{16^{\frac{1}{3}} - 4 \cdot 128^{\frac{1}{3}} + 3 \cdot 250^{\frac{1}{3}}}{2^{\frac{1}{3}}}$

d) $\left(16^{\frac{1}{3}} - 4 \cdot 128^{\frac{1}{3}} + 3 \cdot 250^{\frac{1}{3}}\right) \cdot 2^{-\frac{1}{3}} =$

$\left[2^{\frac{4}{3}} - 4 \cdot 2^{\frac{7}{3}} + 3 \cdot (2 \cdot 5^3)^{\frac{1}{3}}\right] \cdot 2^{-\frac{1}{3}} =$

$\left[2^{\frac{4}{3}} - 4 \cdot 2^{\frac{7}{3}} + 3 \cdot 2^{\frac{1}{3}} \cdot 5^{\frac{3}{3}}\right] \cdot 2^{-\frac{1}{3}} =$

$\left[2^{\frac{4}{3}} - 4 \cdot 2^{\frac{7}{3}} + 15 \cdot 2^{\frac{1}{3}}\right] 2^{-\frac{1}{3}} =$

$2^1 - 4 \cdot 2^2 + 15 \cdot 2^0 =$

$2 - 16 + 15 = 1$

d) $\left(\frac{16}{2}\right)^{\frac{1}{3}} - 4 \left(\frac{128}{2}\right)^{\frac{1}{3}} + 3 \left(\frac{250}{2}\right)^{\frac{1}{3}} =$

$8^{\frac{1}{3}} - 4 \cdot 64^{\frac{1}{3}} + 3 \cdot 125^{\frac{1}{3}} =$

$2 - 16 + 15 = 1$

4.1.17 Simplifier les expressions suivantes et écrivez-les sans fraction :

a) $u^{4/3}u^{-3/2}u^{1/6}$

b) $(a^{-2/3}b^{-1}c^2)^{-3/2} \cdot (a^{-1/2}b^{1/3}c)^{-2}$

c) $\left(\frac{x^{-2/3}y^{3/4}}{x^{5/2}y^{2/3}}\right)^{1/5} \div \left(\frac{x^4y^{-2}}{x^{1/3}y^{-2/5}}\right)^{2/3}$

$$\frac{1}{x^p} = x^{-p}$$

$$c)\ \left[\ x^{-\frac{2}{3}} \cdot x^{-\frac{5}{2}} \cdot y^{\frac{3}{4}} \cdot y^{-\frac{2}{3}} = x^{\frac{-19}{6}} \cdot y^{\frac{1}{12}}\right.$$

$$\left[\ x^{4} \cdot x^{-\frac{1}{3}} \cdot y^{-2} \cdot y^{\frac{2}{5}} = x^{\frac{11}{3}} \cdot y^{-\frac{8}{5}}\right.$$

$$\left(x^{-\frac{19}{6}} \cdot y^{\frac{1}{12}}\right)^{\frac{1}{5}} \cdot \left(x^{\frac{11}{3}} \cdot y^{-\frac{8}{5}}\right)^{-\frac{2}{3}} =$$

$$x^{-\frac{19}{30}} \cdot y^{\frac{1}{60}} \cdot x^{-\frac{22}{9}} \cdot y^{\frac{16}{15}} = \underline{\underline{x^{\frac{-277}{90}} \cdot y^{\frac{13}{12}}}}$$

$$-\frac{19}{30} - \frac{22}{9} = \frac{-57-220}{90} = -\frac{277}{90}$$

$$\frac{1}{60} + \frac{16}{15} = \frac{1+64}{60} = \frac{65}{60} = \frac{13}{12}$$

Ex et log

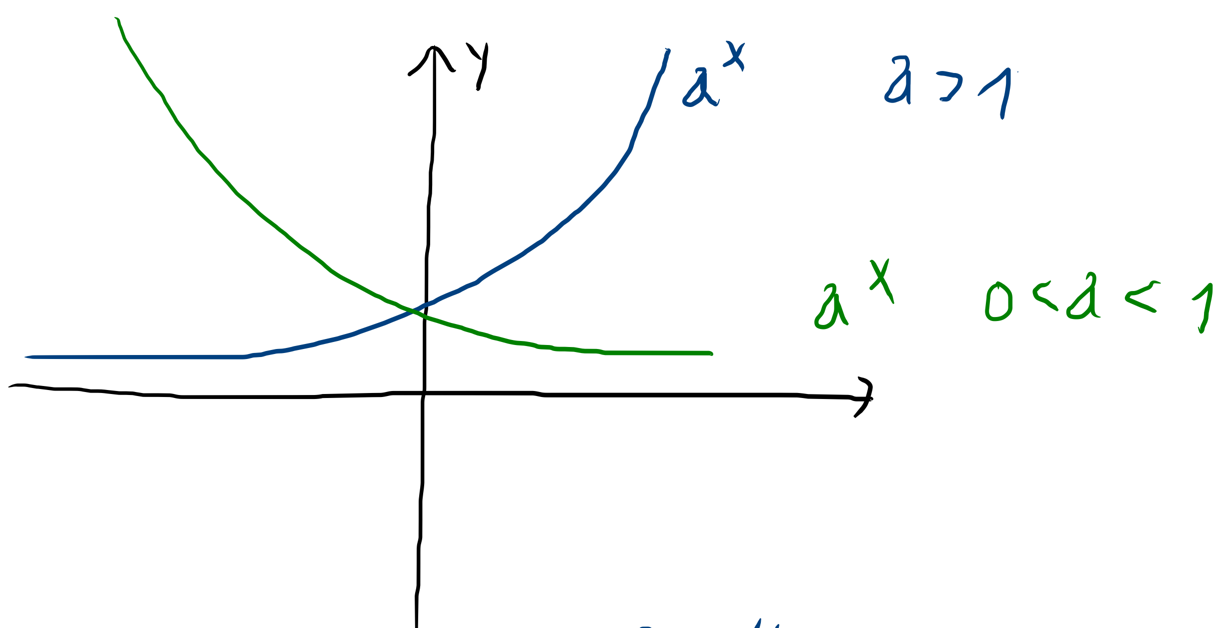
$$\sin(x) = \sin(y)$$
$$\not\Downarrow$$
$$x = y$$

4.2.1 Résoudre les équations ci-dessous :

a) $7^{x+6} = 7^{3x+4}$

b) $6^{7-x} = 6^{2x+1}$

$$\boxed{\text{Soit } a \in \mathbb{R}_+^* - \{1\} \quad a^x = a^y \iff x = y}$$



a) $7^{x+6} = 7^{3x+4} \iff x + 6 = 3x + 4$

$$x = 1$$

$$S = \{1\}$$

d) $9^{(x^2)} = 3^{3x+2}$

e) $2^{-100x} = 0,5^{x-4}$

f) $(\frac{1}{4})^{6-x} = 4$

$0,5 = 2^{-1}$

$2^{-100x} = 2^{-1\cdot(x-4)}$

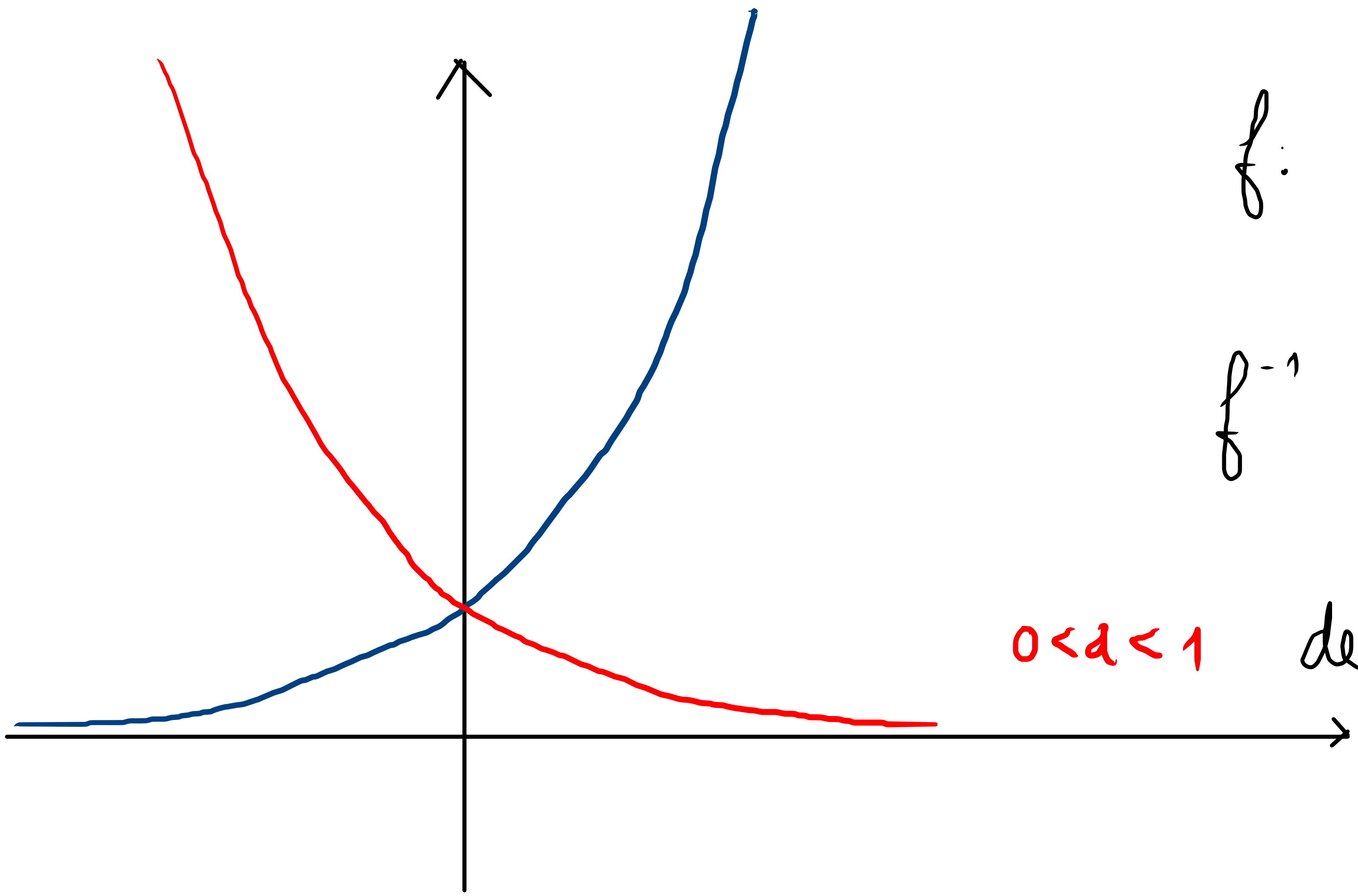
$4^{-1\cdot(6-x)} = 4^{1}$

d) $9^{x^2} = 3^{3x+2}$

$(3^2)^{x^2} = 3^{3x+2}$

$3^{2x^2} = 3^{3x+2} \Rightarrow 2x^2 = 3x+2$

La fonction exp



$a > 1$ croissante

$0 < a < 1$ décroissante

$$f : \mathbb{R} \to \mathbb{R}_+^* \qquad \text{bij}$$
$$x \mapsto a^x$$

$$f^{-1} \quad \mathbb{R}_+^* \to \mathbb{R}$$
$$x \mapsto \log_a(x)$$

$$a \in \mathbb{R}_+^* - \{1\}$$

$$\boxed{a^x = y \iff x = \log_a(y)}$$

- $\log_2(8) = 3 \Leftrightarrow 2^3 = 8$

- $\log_2(1024) = 10$
- $\log_5(25) = 2$
- $\log_{10}(0,1) = -1$
- $\log_3(81) = 4$

4.2.1 et 4.2.2

$\log_2(9) = ?$

$\log_{10}(9) = \log(9)$ $\boxed{\text{TI}}$

$\log_e(9) = \ln(9)$ $\boxed{\text{TI}}$

$\ln(e^2) = \log_e(e^2) = 2$

4.2.1

$$(x^p)^q = x^{pq}$$

$$(x^m)^n = x^{mn}$$

k) $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

$$3^{4x+2} - 36 \cdot 3^{2x+1} + 243 = 0$$

$$\left(3^{2x+1}\right)^2 - 36 \cdot 3^{2x+1} + 243 = 0$$

changement de variable : $3^{2x+1} = t \quad , t \in \mathbb{R}_+^*$

$$t^2 - 36t + 243 = 0$$

$$(t - 27)(t - 9) = 0$$

- $t = 9 \Rightarrow 3^{2x+1} = 3^2 \Rightarrow 2x+1 = 2$
  $\Rightarrow \underline{x = \frac{1}{2}}$
- $t = 27 \Rightarrow 3^{2x+1} = 3^3 \Rightarrow 2x+1 = 3$
  $\underline{x = 1}$

$$S = \left\{ 1/2 \, ; \, 1 \right\}$$

Propriétés des log

Soit $a \in \mathbb{R}_+^* - \{1\}$

$$\log_a(x) = u \iff a^u = x$$
$$\log_a(y) = v \iff a^v = y$$

1) $\log_a(x \cdot y) = \log_a(a^u \cdot a^v) = \log_a(a^{u+v}) = u + v$

$$= \log_a(x) + \log_a(y)$$

2) $\log_a\left(\frac{x}{y}\right) = \log_a\left(\frac{a^u}{a^v}\right) = \log_a\left(a^{u-v}\right) = u - v$

$$= \log_a(x) - \log_a(y)$$

3) $\log_a\left(\frac{1}{y}\right) = -\log_a(y)$

4) $\log_a(x^n) = \log_a\left((a^u)^n\right) = \log_a\left(a^{un}\right)$

$$= u \cdot n = n \cdot \log_a(x)$$

$$6 = \log_2(64) = \log_2(32 \cdot 2) = 5 + 1$$

$$\log_2(32) = 5$$

$$\log_2(2) = 1$$

$$\log_2(1) = 0$$

$$\log_2(3) = \boxed{\text{TI}} \qquad \frac{\log(3)}{\log(2)}$$

$$\log_a\left(x^{\frac{1}{n}}\right) = \frac{\log_a(y)}{n}$$

$$\log\left(\sqrt{2}\right) = \log\left(2^{\frac{1}{2}}\right) = \frac{1}{2}\log(2)$$