

2.5.4

08.01.20

$$\text{d)} \lim_{x \rightarrow 0} \frac{x^2}{\sqrt{x^2 + 1} - 1}$$

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{x^2}{\sqrt{x^2 + 1} - 1} \stackrel{\text{ind}}{=} \lim_{x \rightarrow 0} \frac{x^2}{\sqrt{x^2 + 1} - 1} \cdot \frac{\sqrt{x^2 + 1} + 1}{\sqrt{x^2 + 1} + 1} \\
 &= \lim_{x \rightarrow 0} \frac{x^2 (\sqrt{x^2 + 1} + 1)}{x^2 + 1 - 1} = \lim_{x \rightarrow 0} \frac{x^2 (\sqrt{x^2 + 1} + 1)}{x^2} \\
 &= 2
 \end{aligned}$$

$$h) \lim_{x \rightarrow -2} \frac{x + \sqrt{x+6}}{x + \sqrt{2-x}} \stackrel{\text{ind}}{=} \lim_{\substack{x \rightarrow -2 \\ "0" \over 0}} \frac{x + \sqrt{x+6}}{x + \sqrt{2-x}} \cdot \frac{x - \sqrt{2-x}}{x - \sqrt{2-x}}$$

$$= \lim_{x \rightarrow -2} \frac{(x + \sqrt{x+6})(x - \sqrt{2-x})}{x^2 + x - 2} \stackrel{\text{ind}}{=} \frac{0}{0}$$

$$= \lim_{x \rightarrow -2} \frac{(x - \sqrt{2-x})(x + \sqrt{x+6})}{(x+2)(x-1)} \cdot \frac{x - \sqrt{x+6}}{x - \sqrt{x+6}}$$

$$= \lim_{x \rightarrow -2} \frac{(x - \sqrt{2-x})(x^2 - x - 6)}{(x+2)(x-1)(x - \sqrt{x+6})}$$

$$= \lim_{x \rightarrow -2} \frac{(x - \sqrt{2-x})(x - 3)(x + 2)}{(x+2)(x-1)(x - \sqrt{x+6})} = \frac{(-2 - 2)(-2 - 3)}{(-2 - 1)(-2 - 2)}$$

$$= \frac{-5}{-3} = \frac{5}{3}$$

$$\text{e) } \lim_{x \rightarrow 5} \frac{x-5}{\sqrt{2x-1}-3} \stackrel{\text{ind}}{=} \lim_{\substack{x \rightarrow 5 \\ 0}} \frac{x-5}{\sqrt{2x-1}-3} \cdot \frac{\sqrt{2x-1}+3}{\sqrt{2x-1}+3}$$

$$= \lim_{x \rightarrow 5} \frac{(x-5)(\sqrt{2x-1}+3)}{2x-1-9} = \lim_{x \rightarrow 5} \frac{(x-5)(\sqrt{2x-1}+3)}{2(x-5)}$$

$$= \frac{6}{2} = 3$$

2.5.5

d) $f(x) = \frac{|x-2|}{x^2 - 3x + 2} \quad x_0 = 2$

$$|x-2| \begin{cases} x-2, & x \geq 2 \\ -x+2, & x < 2 \end{cases}$$

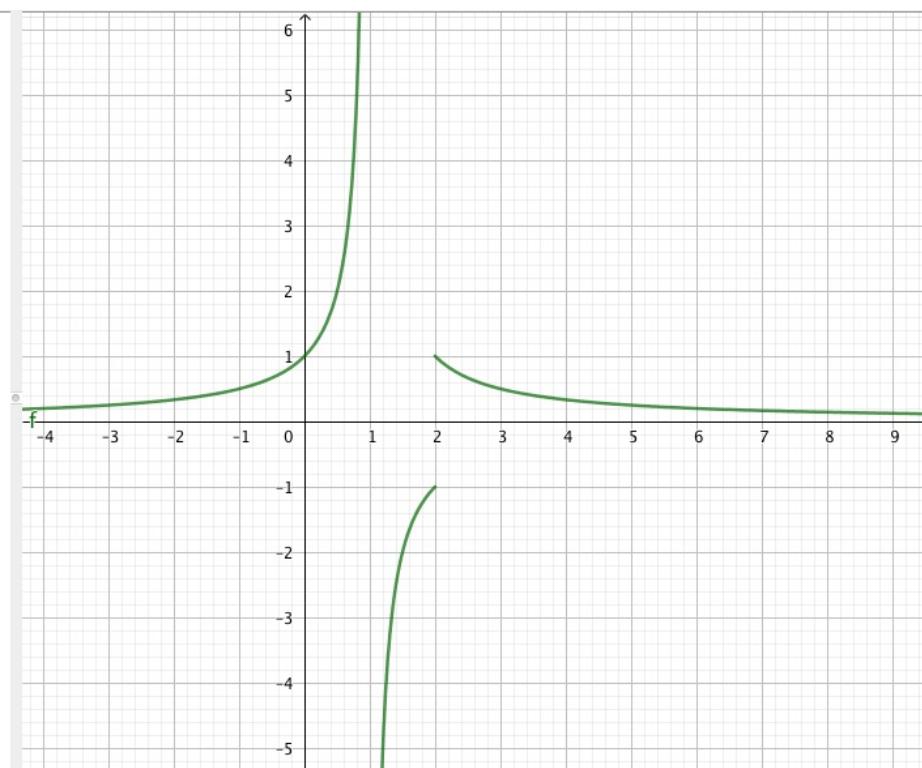
$$\lim_{x \rightarrow 2} f(x) = \text{"indéterminée"}$$

"0/0"

A gauche : $\lim_{x \rightarrow 2^-} \frac{-x+2}{(x-2)(x-1)} = \lim_{x \rightarrow 2^-} \frac{-(x-2)}{(x-2)(x-1)} = \frac{-1}{1} = -1$

A droite : $\lim_{x \rightarrow 2^+} \frac{x-2}{(x-2)(x-1)} = \frac{1}{1} = 1$

$\lim_{x \rightarrow 2} f(x)$ n'existe pas.



2.5.6 Utiliser le théorème « des deux gendarmes » pour déterminer :

$$\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) \quad \text{et} \quad \lim_{x \rightarrow 0} \sqrt{x} \cos\left(\frac{1}{x}\right)$$

Si $g(x) \leq f(x) \leq h(x)$ et $x \in]\alpha - \delta, \alpha + \delta[$ et que

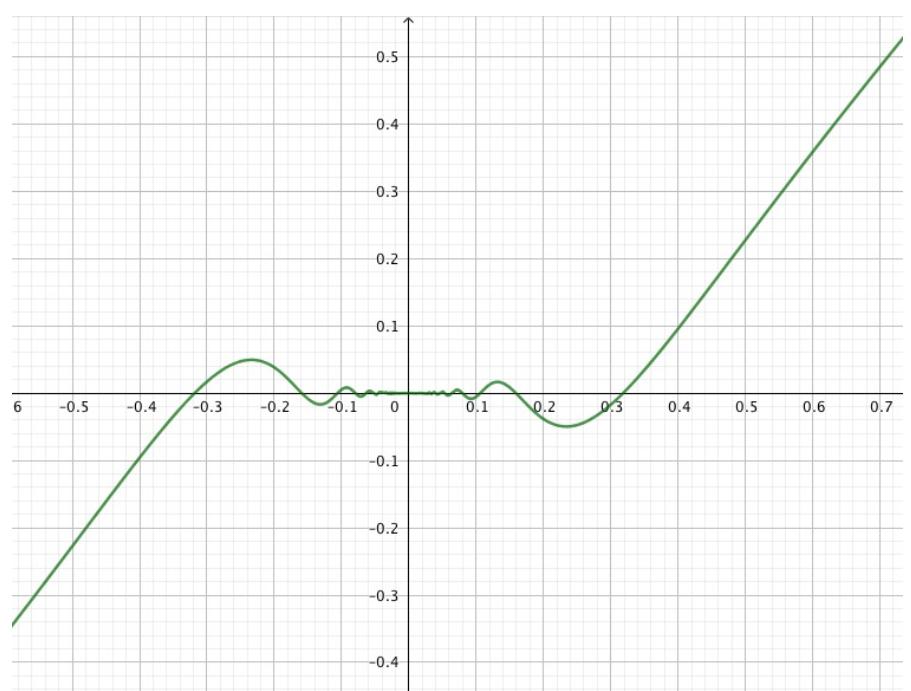
$$\lim_{x \rightarrow \alpha} g(x) = \lim_{x \rightarrow \alpha} h(x) = L, \text{ alors } \lim_{x \rightarrow \alpha} f(x) = L$$

2)

$$-x^2 \leq x^2 \cdot \underbrace{\sin\left(\frac{1}{x}\right)}_{-1 \leq \sin\left(\frac{1}{x}\right) \leq 1} \leq x^2$$

Comme $\lim_{x \rightarrow 0} -x^2 = \lim_{x \rightarrow 0} x^2 = 0$, donc

$$\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

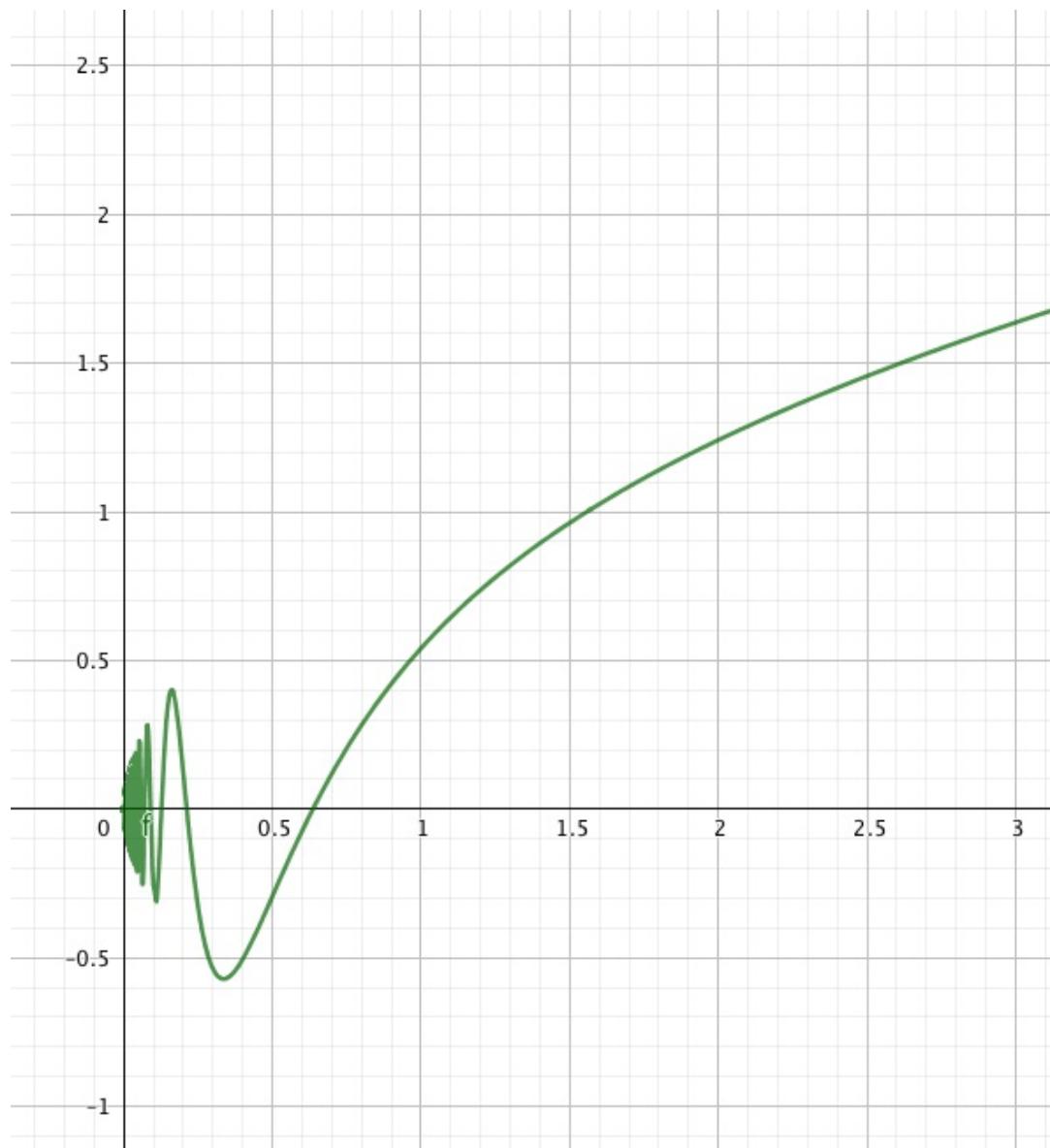


b)

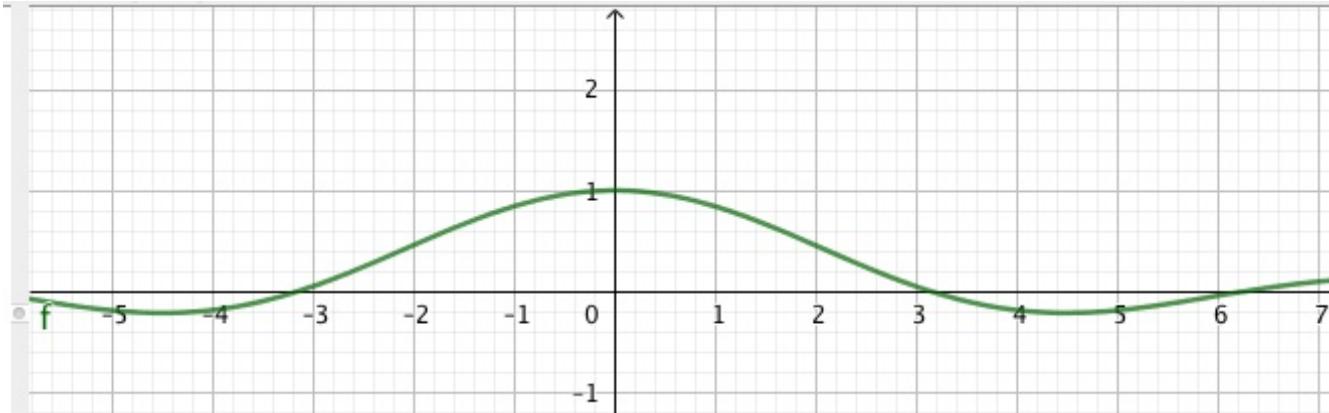
$x > 0$

$$0 \leq \sqrt{x} \cos\left(\frac{1}{x}\right) \leq \sqrt{x}$$

$$\lim_{x \rightarrow 0^+} \sqrt{x} \cos\left(\frac{1}{x}\right) = 0$$



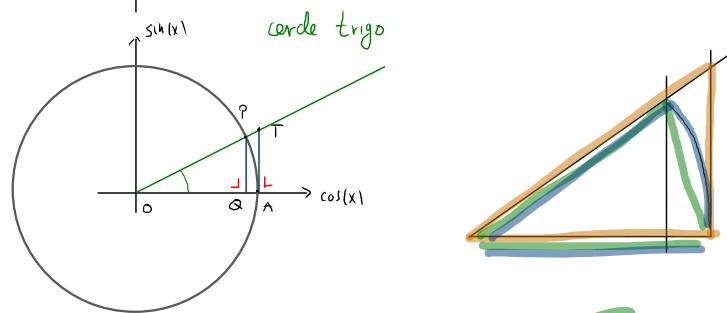
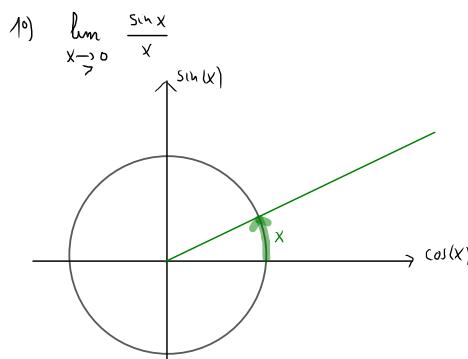
Calcul de $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$



x	$\sin x / x$
0.1	0.998334166
0.01	0.999983333
0.001	0.999999833
0.0001	0.999999998
-0.1	0.998334166
-0.01	0.999983333
-0.001	0.999999833
-0.0001	0.999999998

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \text{par l'expérience}$$

$$\frac{\sin(x)}{x} \approx \frac{x}{x} = 1 \quad x \text{ petit}$$



Aire de $\triangle OPA$: $\frac{1}{2} OA \cdot QP = \frac{1}{2} \cdot 1 \cdot \sin(x) = \frac{\sin(x)}{2}$

Aire de $\triangle OAT$: $\frac{1}{2} OA \cdot AT = \frac{1}{2} \cdot 1 \cdot \tan(x) = \frac{\tan(x)}{2}$

Aire du secteur OPA : $\frac{x}{2}$

$$\frac{\sin(x)}{2} \leq \frac{x}{2} \leq \frac{\tan(x)}{2} \quad x > 0$$

$$\Rightarrow \sin(x) \leq x \leq \tan(x) \quad 4 \leq 9 \leq 13$$

$$\Rightarrow \frac{1}{\sin(x)} \geq \frac{1}{x} \geq \frac{1}{\tan(x)} \quad \frac{1}{4} \geq \frac{1}{9} \geq \frac{1}{13}$$

$$\Rightarrow \frac{1}{\sin(x)} \geq \frac{1}{x} \geq \frac{\cos(x)}{\sin(x)} \quad \sin(x) > 0$$

$$\frac{\sin(x)}{\sin(x)} \geq \frac{\sin(x)}{x} \geq \frac{\cos(x) \cdot \sin(x)}{\sin(x)}$$

$$1 \geq \frac{\sin(x)}{x} \geq \cos(x)$$

Comme $\lim_{x \rightarrow 0^+} 1 = \lim_{x \rightarrow 0^+} \cos(x) = 1$, alors

$$\boxed{\lim_{x \rightarrow 0^+} \frac{\sin(x)}{x} = 1}$$

Prenons $x < 0$, donc $-x > 0$ et on a

$$\frac{\sin(-x)}{-x} = \frac{-\sin(x)}{-x} = \frac{\sin(x)}{x}$$

Finlement

$$\boxed{\lim_{x \rightarrow 0^-} \frac{\sin(x)}{x} = 1}$$

2.5.8 Calculer les limites suivantes :

$$a) \lim_{x \rightarrow 0} \frac{\sin(2x)}{x} = \lim_{x \rightarrow 0} \underbrace{\frac{\sin(2x)}{(2x)}}_1 \cdot 2 = 2$$

$$\lim_{t \rightarrow 0} \frac{\sin(t)}{t} = \lim_{\varepsilon \rightarrow 0} \frac{\sin(100\varepsilon)}{100\varepsilon} = 1$$

$$e) \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1} = \lim_{t \rightarrow 0} \frac{\sin(t)}{t} = 1$$

$$t = x - 1$$

$$x = t + 1$$

