

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

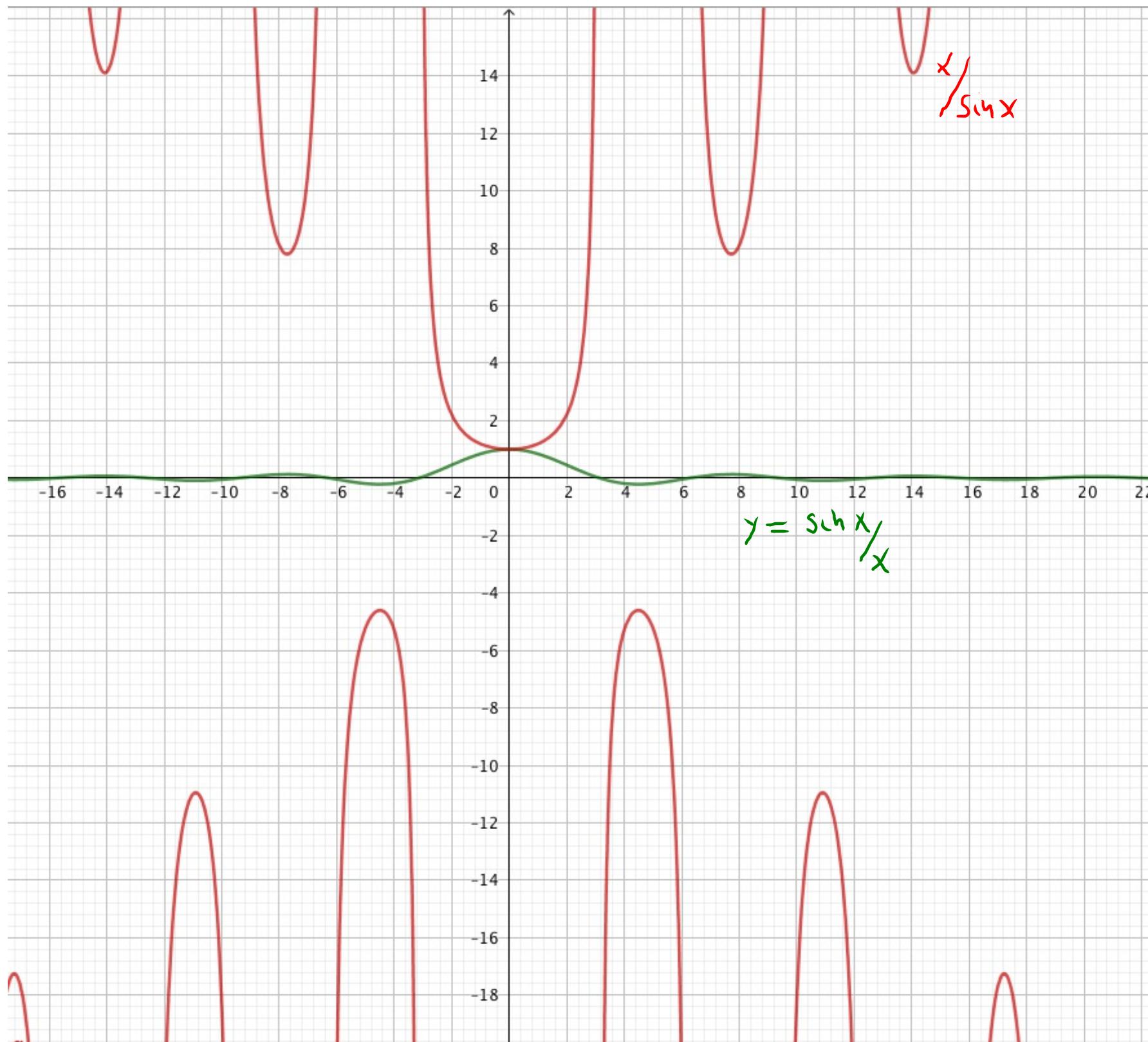
2.5.8

$$\left. \begin{aligned} \lim_{x \rightarrow 2} f(x) &= L_1 \\ \lim_{x \rightarrow 2} g(x) &= L_2 \\ \lim_{x \rightarrow 2} f(x) \cdot g(x) &= L_1 \cdot L_2 \end{aligned} \right\}$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(2x)} = \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \cdot \frac{2x}{\sin(2x)} \cdot \frac{3}{2}$$

$$= \frac{3}{2} \left[\underbrace{\lim_{x \rightarrow 0} \frac{\sin(3x)}{3x}}_{=1} \cdot \underbrace{\lim_{x \rightarrow 0} \frac{2x}{\sin(2x)}}_{=1} \right] = \frac{3}{2}$$

$$1 = \lim_{x \rightarrow 0} \frac{\sin(x)}{x} = \lim_{x \rightarrow 0} \frac{1}{\frac{x}{\sin(x)}} = \frac{1}{\lim_{x \rightarrow 0} \frac{x}{\sin(x)}} \approx 1$$



2.5.8

g) $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{\sin^2(x)}$

h) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos(x)}{x - \frac{\pi}{2}}$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{1 - \cos^2(x)} = \lim_{x \rightarrow 0} \frac{1 - \cancel{\cos}(x)}{\cancel{(1 - \cos(x))}(1 + \cos(x))} =$$

$$= \lim_{x \rightarrow 0} \frac{1}{1 + \cos(x)} = \frac{1}{2}$$

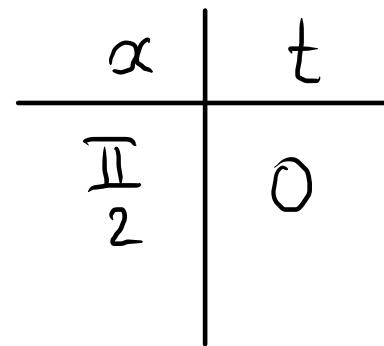
$\hookrightarrow x \text{ tend vers } 0 \Rightarrow \cos(x) = 1$

Formulaire

h) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos(x)}{x - \frac{\pi}{2}} = \lim_{t \rightarrow 0} \frac{\cos(t + \frac{\pi}{2})}{t} \stackrel{\text{ind}}{=} \lim_{t \rightarrow 0} \frac{-\sin(t)}{t} = -1$

Changement de variables

$$t = x - \frac{\pi}{2}$$



2.5.9 En ampliant chaque fraction par $\cos(x) + 1$, montrer que :

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x} = 0 \quad \text{et} \quad \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x} \cdot \frac{\cos(x) + 1}{\cos(x) + 1} = \lim_{x \rightarrow 0} \frac{-\sin^2(x)}{x(1 + \cos(x))}$$

$$= \lim_{x \rightarrow 0} \left[\frac{-\sin(x)}{x} \cdot \frac{\sin(x)}{1 + \cos(x)} \right]$$

$$= - \lim_{x \rightarrow 0} \frac{\sin(x)}{x} \cdot \lim_{x \rightarrow 0} \frac{\sin(x)}{1 + \cos(x)} = 0$$

2.5.10 Trouver une fonction f non définie en a telle que $\lim_{x \rightarrow a} f(x) = b$:

a) $a = 2$ et $b = 3$

b) $a = -1$ et $b = 7$

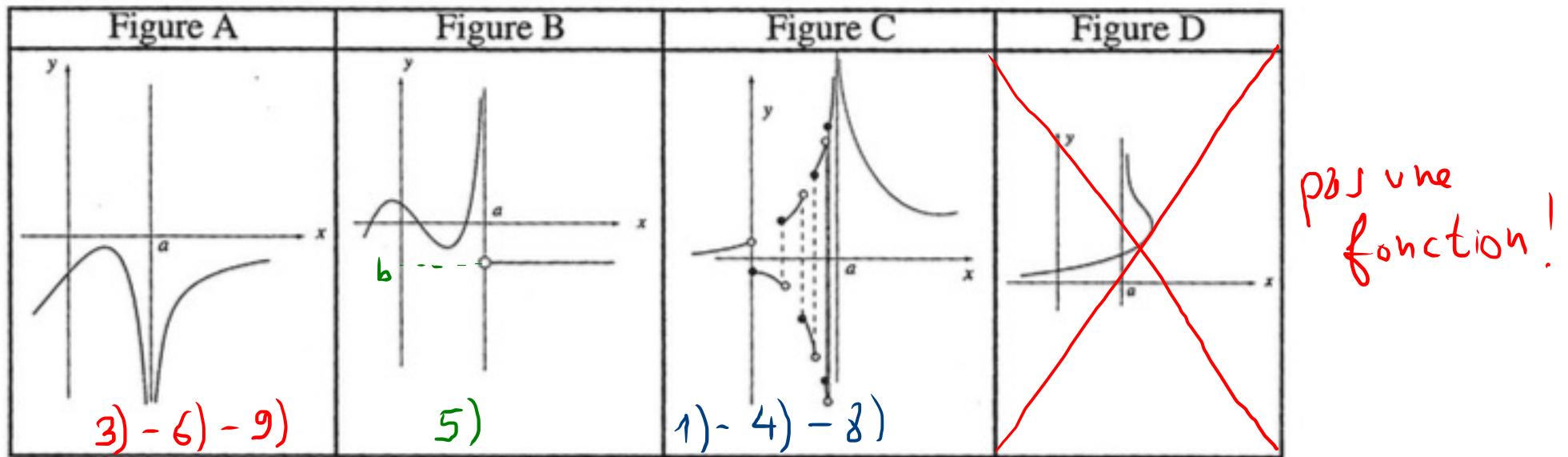
2) $\lim_{x \rightarrow 2} f(x) = 3$ $ED(f) = \mathbb{R} - \{2\}$

$$\lim_{x \rightarrow 2} \frac{3x-6}{x-2} = \lim_{x \rightarrow 2} \frac{\cancel{x-2}}{\cancel{x-2}} \cdot 3$$

b) $\lim_{x \rightarrow -1} \frac{7x+7}{x+1} = 7$

ou $\lim_{x \rightarrow -1} \frac{(7x+7)(2x+3)}{x+1} = 7$

2.5.13 Dire pour chacune des quatre figures ci-dessous quelles sont les notations autorisées parmi 1), 2), ..., 9) :



$$\lim_{x \rightarrow a} f(x) = \begin{cases} 1) & \infty \\ 2) & +\infty \\ 3) & -\infty \end{cases}$$

$$\lim_{x \leftarrow a} f(x) = \begin{cases} 4) & \infty \\ 5) & +\infty \\ 6) & -\infty \end{cases}$$

$$\lim_{x \rightarrow a} f(x) = \begin{cases} 7) & \infty \\ 8) & +\infty \\ 9) & -\infty \end{cases}$$

B $\lim_{x \rightarrow a} f(x) = b$

2.5.8 - 2.5.9 - 2.5.11

Figure C

