

11.09.19

1.2.10 Déterminer sous forme algébrique les racines carrées complexes des nombres ci-dessous :

a) 1

d) -9

b) i

e) 3 + 4i

c) -i

f) -5 + 12i = z

$$w = \sqrt{a^2 + b^2}$$

$$|z| = \sqrt{(-5)^2 + 12^2} = 13$$

f) $w = a + bi$ tel $w^2 = z$ ($|w|^2 = |z|$)

$$w^2 = a^2 - b^2 + 2abi$$

$$\begin{cases} a^2 - b^2 = -5 & \text{partie réelle} \\ 2ab = 12 & \text{partie imaginaire} \\ a^2 + b^2 = 13 & \text{module} \end{cases}$$

$$\Rightarrow \begin{cases} a^2 - b^2 = -5 \\ a^2 + b^2 = 13 \\ ab = 6 \end{cases} \begin{array}{l} \cdot 1 \\ \cdot 1 \end{array} \left| \begin{array}{l} b^2 \\ a^2 \end{array} \right. \begin{array}{l} \cdot (-1) \\ \cdot 1 \end{array} \Rightarrow \begin{cases} 2a^2 = 8 \\ 2b^2 = 18 \\ ab = 6 \end{cases} \Rightarrow \begin{cases} a = \pm 2 \\ b = \pm 3 \\ ab = 6 \end{cases}$$

Les racines sont $2 + 3i$, $-2 - 3i$

$$\sqrt{3 - 3i}$$

 $\sqrt{\quad}$

$$\sqrt{9} = \cancel{3}$$

$$\sqrt{4} = 2$$

On n'écrit pas $\sqrt{-5 + 12i}$ $\begin{cases} 2 + 3i \\ -2 - 3i \end{cases}$

Equations dans \mathbb{C}

$$1) \quad a_n X^n + a_{n-1} X^{n-1} + \dots + a_0 = 0, \quad a_k \in \mathbb{R}, k \in \{0, \dots, n\}$$

Si $z \in \mathbb{C}$ est solution, alors \bar{z} est aussi solution

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$$c) \quad 2z^2 + 10z + 17 = 0$$

$$\Delta = -36$$

Trouver $w = a + bi$ tq $w^2 = -36 \Rightarrow w = \pm 6i$

$$z = \frac{-10 \pm 6i}{4} = -\frac{5}{2} \pm \frac{3}{2}i$$

$$z_1 = -\frac{5}{2} - \frac{3}{2}i \quad \text{et} \quad z_2 = -\frac{5}{2} + \frac{3}{2}i$$

$$ax^2 + bx + c = 0$$

$$\Delta = b^2 - 4ac$$

$$\Delta < 0$$

$$\Delta = 0$$

$$\Delta > 0$$

$$\frac{-b}{2a}$$

$$\frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\text{a) } z^2 = 25$$

$$\text{b) } z^2 = -4$$

$$\text{a) } z^2 - 25 = 0$$

$$\Delta = 100 = 10^2$$

$$\frac{-0 \pm 10}{2} = \pm 5$$

$$\alpha > 0 \quad z^2 = \alpha$$

$$z = \pm \sqrt{\alpha}$$

$$\text{b) } z^2 = -4$$

$$z = \pm 2i$$

$$\alpha < 0 \quad z^2 = \alpha$$

$$z = \pm \sqrt{|\alpha|} i$$

$$d) \quad d) \quad z^2 + 3z - 5 = 0$$

$$\Delta = 9 + 20 = 29$$

$$\text{solutions : } \frac{-3 \pm \sqrt{29}}{2}$$

$$e) \quad z^2 - 3(1+i)z + 6 + 7i = 0$$

$$\begin{aligned} \Delta &= (-3(1+i))^2 - 4 \cdot 1 \cdot (6 + 7i) \\ &= -24 - 10i \end{aligned}$$

$$w = a + bi \quad \text{tel} \quad w^2 = -24 - 10i$$

$$\text{Par calcul} \quad w_1 = 1 - 5i \quad \text{et} \quad w_2 = -1 + 5i \quad \dots$$

1.3.2 Décomposer dans $\mathbb{R}[z]$ et $\mathbb{C}[z]$ les polynômes ci-dessous:

a) $z^4 - 1 = p$

d) $z^6 - 1$

a) Dans $\mathbb{R}[z]$

$$p = (z^2 - 1)(z^2 + 1) = \underline{(z-1)(z+1)(z^2 + 1)}$$

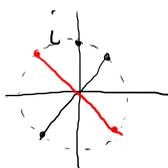
Dans $\mathbb{C}[z]$

$$p = \underline{(z-1)(z+1)(z-i)(z+i)}$$

b) $p = z^4 + 1$

dans $\mathbb{C}[z]$ $p = (z^2 - i)(z^2 + i)$

calculer les racines carrées de $-i$ et i



① $i = [1; \frac{\pi}{2}]$ $w = [r, \theta]$, $w^2 = [r^2, 2\theta]$

$$\begin{cases} r^2 = 1 \\ 2\theta = \frac{\pi}{2} + 2k\pi \end{cases} \Rightarrow \begin{cases} r = 1 \\ \theta = \frac{\pi}{4} + k\pi \end{cases}$$

Les racines carrées de i : $[1; \frac{\pi}{4}]$ et $[1; \frac{5\pi}{4}]$

$z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ et $z_2 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

② $-i = [1; \frac{3\pi}{2}]$

$$\begin{cases} r^2 = 1 \\ 2\theta = \frac{3\pi}{2} + 2k\pi \end{cases} \Rightarrow \begin{cases} r = 1 \\ \theta = \frac{3\pi}{4} + k\pi \end{cases}$$

$\bar{z}_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ $\bar{z}_1 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$$z^4 + 1 = \left(z - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right) \left(z - \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\right) \left(z - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\right) \left(z - \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right)$$

$$z^4 + 1 = (z^2 - \sqrt{2}z + 1) (z^2 + \sqrt{2}z + 1)$$