

Soit  $f$  une fonction différentiable. Si  $x_0 \in ED(f)$  et est tel que  $f'(x_0) = 0$ , on dit que  $x_0$  est un point stationnaire de  $f$ .

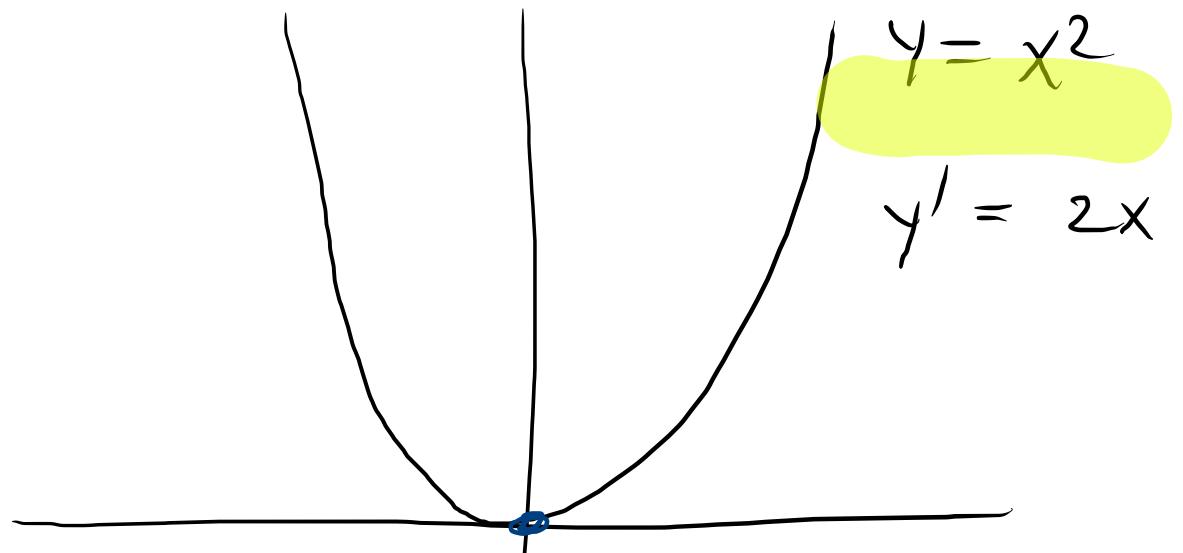
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Recherche des points où une fonction atteint ses extrêmes

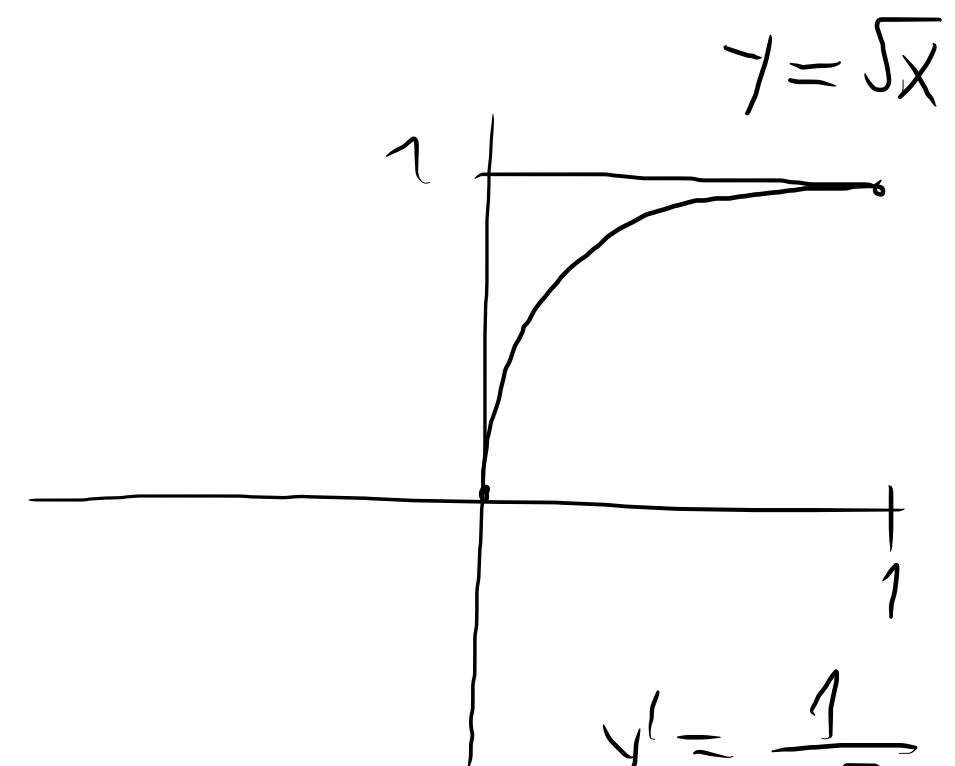
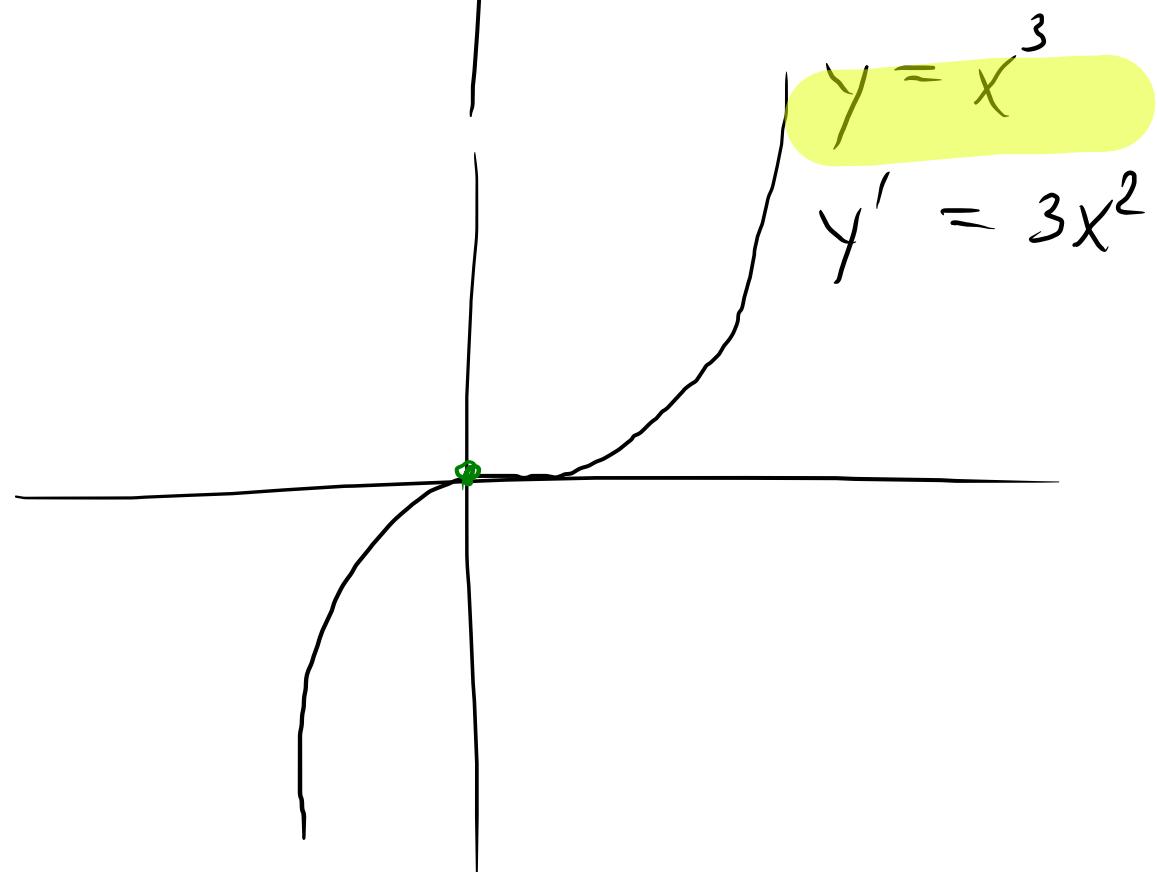
$f: [a,b] \rightarrow \mathbb{R}$  une fonction.

Les extrêmes de  $f$  se trouvent forcément parmi les points suivants :

- $a$  et  $b$
- les points stationnaires de  $f$
- les points de l'intervalle qui n'appartiennent pas à  $ED(f')$



$$[0, 1]$$



$$ED(f') = [0; 1]$$

## Exemple

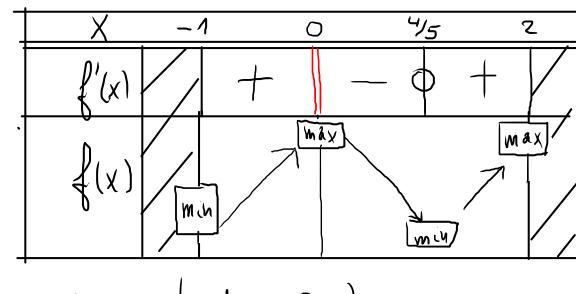
Chercher les extrema de  $f: [-1, 2] \rightarrow \mathbb{R}$

$$x \mapsto (x-2) \cdot \sqrt[3]{x^2}$$

$$\begin{aligned} f'(x) &= (x-2)\left(\frac{2}{3}x^{-\frac{1}{3}}\right) + x^{\frac{2}{3}} = \\ &= \frac{2x-4}{3\sqrt[3]{x^2}} + \sqrt[3]{x^2} = \\ &= \frac{2x-4 + 3x^{\frac{2}{3}} \cdot x^{\frac{1}{3}}}{3\sqrt[3]{x}} = \\ &= \frac{2x-4 + 3x}{3\sqrt[3]{x}} = \frac{5x-4}{3\sqrt[3]{x}} \end{aligned}$$

$$ED(f') = [-1; 0[ \cup ]0; 2]$$

Tableau des variations de  $f$ :



$$\min (-1; -3)$$

$$\max (0; 0)$$

$$\min \left(\frac{4}{5}; -1,03\right)$$

$$\max (2; 0)$$

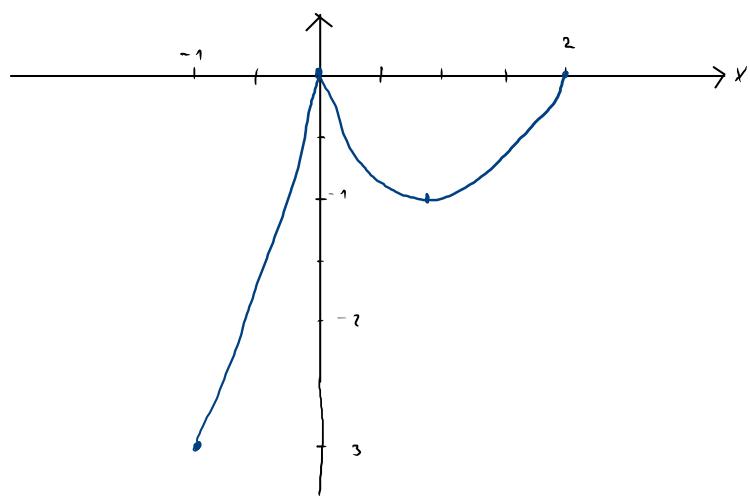
$$f(-1) = -3 \cdot \sqrt[3]{(-1)^2}$$

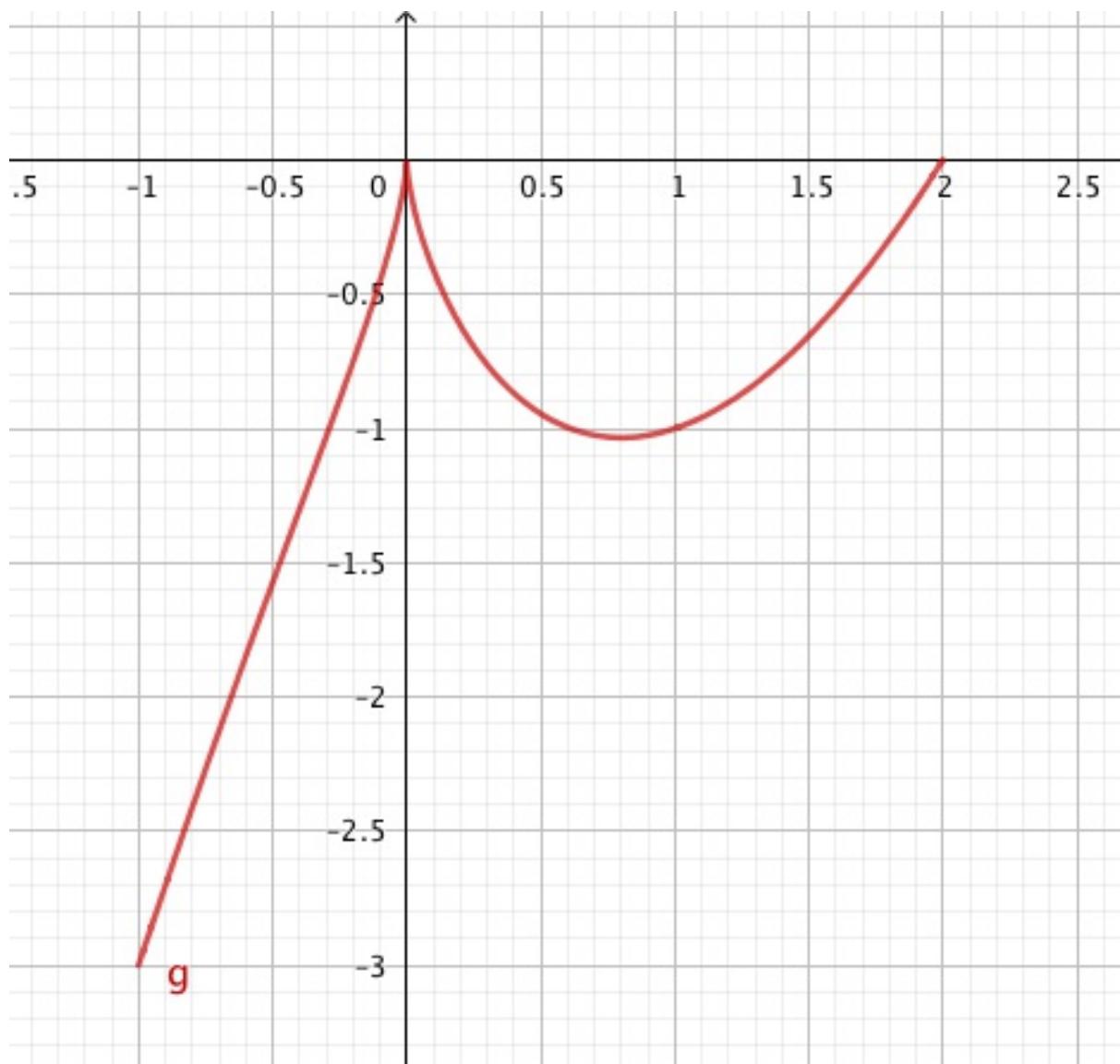
$$= -3$$

$$f\left(\frac{4}{5}\right) = \left(\frac{4}{5}-2\right) \sqrt[3]{\frac{16}{25}}$$

$$= -\frac{6}{5} \cdot \sqrt[3]{\frac{16}{25}}$$

$$\approx -1,03$$

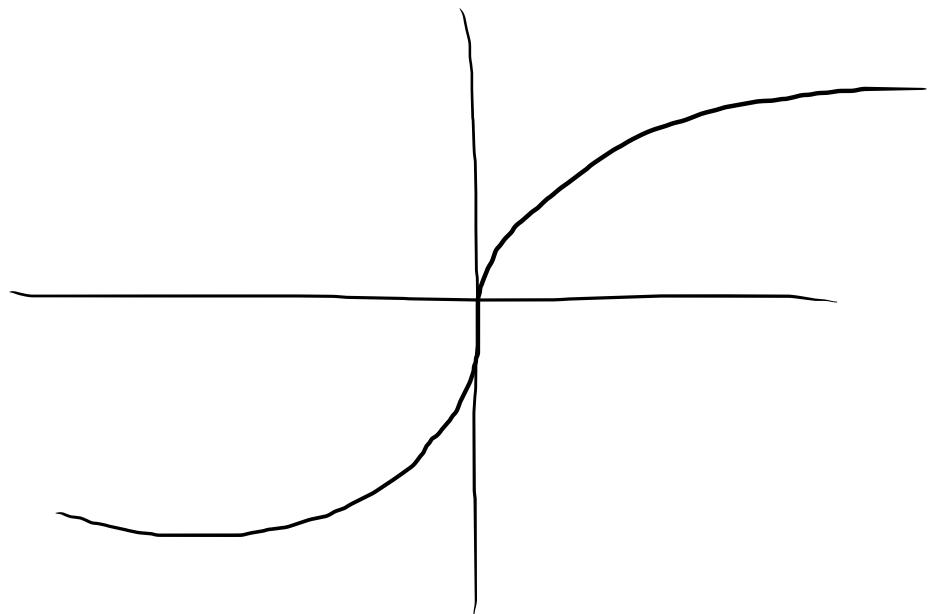
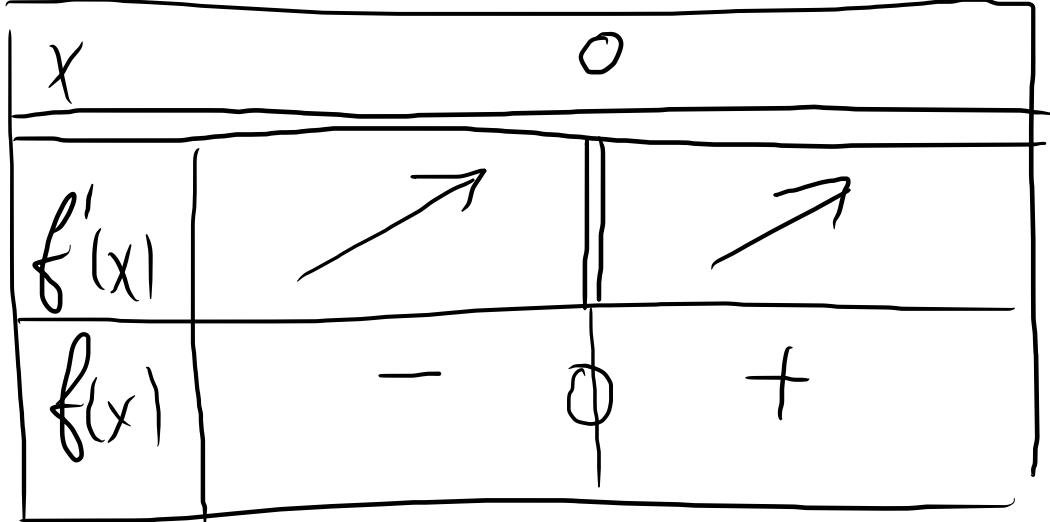




$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$ED(f) = \mathbb{R}$$

$$f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$



h)  $f(x) = \sin(x)(1 + \cos(x))$ , sur  $[0; 2\pi]$

$$ED(f) = [0, 2\pi]$$

$$ED(f') = [0, 2\pi]$$

$$\begin{aligned} f'(x) &= \cos(x)(1 + \cos(x)) + \sin(x)(-\sin(x)) \\ &= \underline{\cos(x)} + \underline{\cos^2(x)} - \underline{\sin^2(x)} \\ &= \cos(x) + \cos^2(x) - (1 - \cos^2(x)) \\ &= 2\cos^2(x) + \cos(x) - 1 \end{aligned}$$

Calculons les zéros de  $f'(x)$ :  $\cos(x) = t$

$$2t^2 + t - 1 = 0$$

$$(2t - 1)(t + 1) = 0$$

$$t = \frac{1}{2} \quad \text{ou} \quad t = -1$$

a)  $\cos(x) = \frac{1}{2} \quad \stackrel{(T1)}{\Rightarrow} \quad x = \frac{\pi}{3}$

$$\begin{cases} x = \frac{\pi}{3} + 2K\pi \\ x = -\frac{\pi}{3} + 2K\pi \end{cases}$$

b)  $\cos(x) = -1 \quad \stackrel{(T1)}{\Rightarrow} \quad x = \pi$

$$x = \pi + 2K\pi$$

entre  $0$  et  $2\pi$ :  $\frac{\pi}{3}, \frac{5\pi}{3}, \pi$

Tableau des variations:

$x$	$0$	$\frac{\pi}{3}$	$\pi$	$\frac{5\pi}{3}$	$2\pi$
$f'(x)$	/	+	0	-	0
$f(x)$	/	max	pl	min	max

$$f'(x) = (\cos(x) + 1)(2\cos(x) - 1)$$

$$f(x) = \sin(x)(1 + \cos(x))$$

$$\begin{array}{ll} \min & (0; 0) \quad \left(\frac{5\pi}{3}; -\frac{3\sqrt{3}}{4}\right) \\ \max & \left(\frac{\pi}{3}; \frac{3\sqrt{3}}{4}\right) \quad (2\pi; 0) \end{array}$$

$$\text{pt: } (\pi; 0)$$

