14.11.19

4.2.7 Résoudre les systèmes d'équations :

a) $\begin{cases} \log(x) + \log(y) & = 2 \\ x + y & = 25 \end{cases}$ b) $\begin{cases} \log(x) - \log(y) & = 1 \\ x\,y & = 2 \end{cases}$

$x, y \in \mathbb{R}_+^*$

a) $\begin{cases} x + y = 25 \\ \log(xy) = \log(100) \end{cases} \Rightarrow \begin{cases} x + y = 25 \\ xy = 100 \end{cases}$

$\Rightarrow \begin{cases} y = -x + 25 \\ x(-x + 25) = 100 \end{cases} \Rightarrow \begin{cases} y = -x + 25 \\ x^2 - 25x + 100 = 0 \end{cases}$

$\begin{cases} y = -x + 25 \\ (x - 5)(x - 20) = 0 \end{cases}$

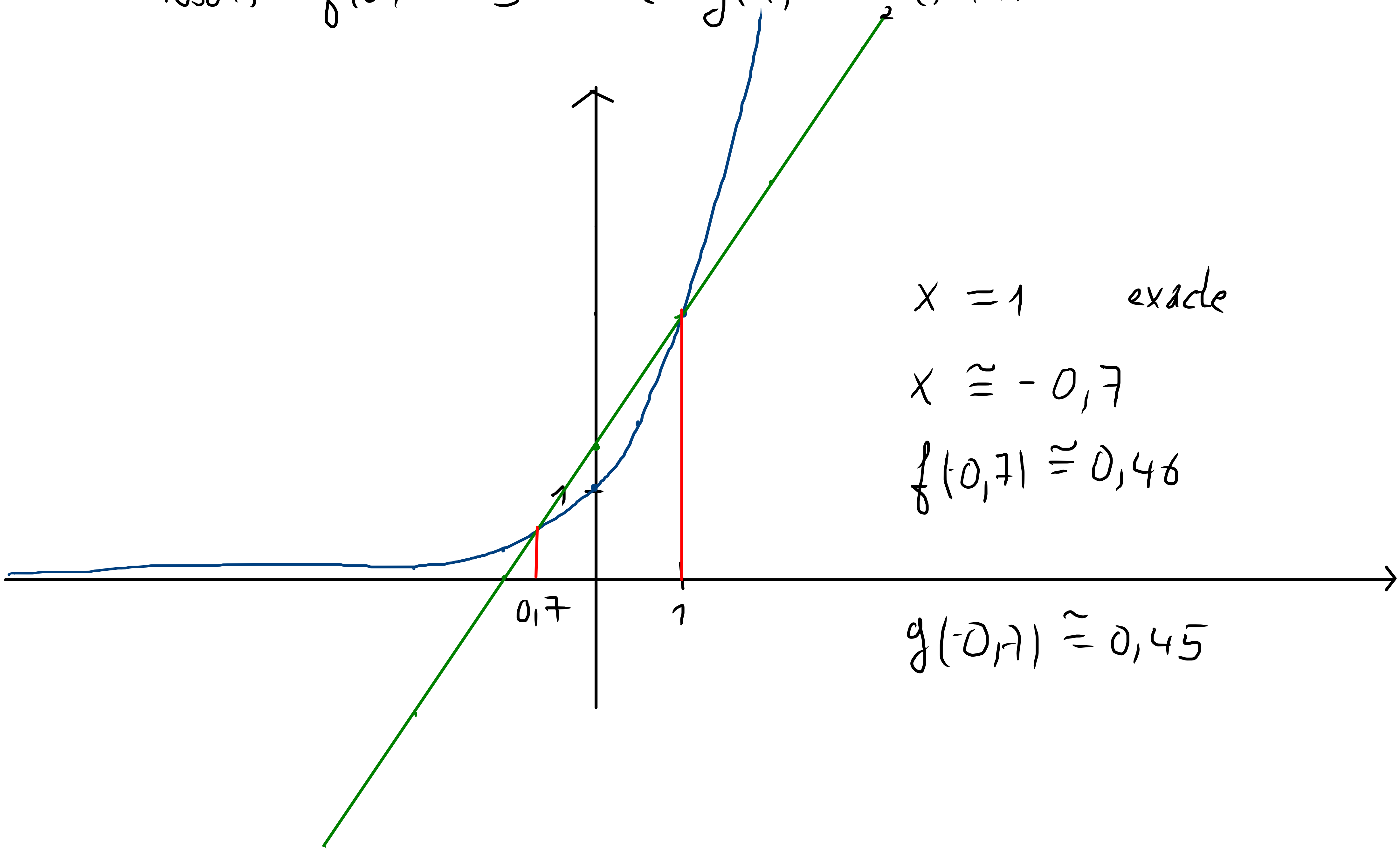
$\begin{cases} x = 5 & , y = 20 \\ x = 20 & y = 5 \end{cases} \Rightarrow S = \{(5;20), (20,5)\}$

4.2.8 Estimer graphiquement les solutions des équations suivantes :

a) $3^x = \frac{3}{2}(x+1)$

b) $x + \log_3(x) = 0$

Posons $f(x) = 3^x$ et $g(x) = \frac{3}{2}(x+1)$



$x = 1$ exacte

$x \cong -0,7$

$f(-0,7) \cong 0,46$

$g(-0,7) \cong 0,45$

4.2.10 Donner l'ensemble de définition des fonctions suivantes :

a) $f(x) = \dfrac{2}{10^x - 9}$

b) $f(x) = \log_7\left(\dfrac{x^2-1}{x+3}\right)$

c) $f(x) = \log(x^3 + 2x^2 - 3)$ Horner

d) $f(x) = \dfrac{1}{\log(x^2-1)}$

a) zéro du dénominateur $10^x = 9 \iff \log(9)$

$ED(f) = \mathbb{R} - \{\log(9)\}$

b) condition : $\dfrac{x^2-1}{x+3} > 0$

Tableau des signes

x		-3		-1		1	
$\frac{x^2-1}{x+3}$	$-$	$\Vert$	$+$	0	$-$	0	$+$

$ED(f) =]-3;-1[\cup]1;+\infty[$

d) ① $\log(x^2-1) \neq 0$

② $x^2 - 1 > 0$

x		-1		1	
x^2-1	$+$	0	$-$	0	$+$

cond ② $x \in]-\infty;-1[\cup]1;+\infty[$

① $\log(x^2-1) = 0 \iff x^2 - 1 = 1$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

$ED(f) =]-\infty;-1[\cup]1;+\infty[- \{-\sqrt{2};\sqrt{2}\}$

$$f(x) = (x-3)^{2}(x+3)^{3}(x-2)^{4}$$

x		-3 i		2 p		3 p	
$f(x)$	$-$	0	$+$	0	$+$	0	$+$

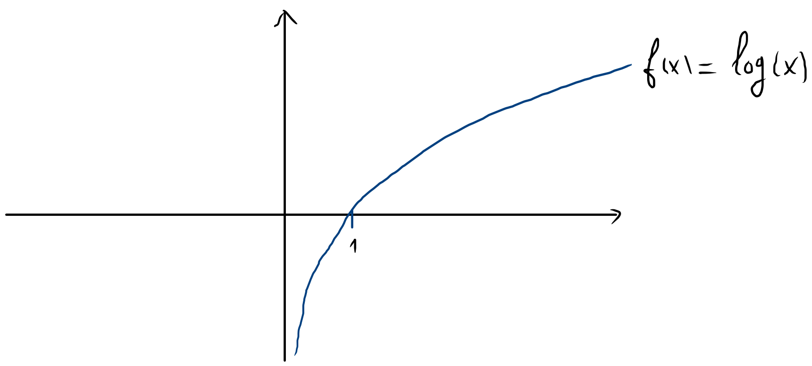
$$\log(x) = y \iff 10^{y} = x$$

4.2.11 Etablir le tableau des signes des fonctions suivantes :

a) $f(x) = \log(-x^2 + 4x + 22)$

b) $f(x) = 12 - 10^{3-x}$

c) $f(x) = \log_2\left(\frac{2x}{x-1}\right)$



a) Recherche de $ED(f)$: signe de $-x^2 + 4x + 22$

x		$2-\sqrt{26}$		$2+\sqrt{26}$	
$-x^2+4x+22$	$-$	0	$+$	0	$-$

$\Delta = 16 + 88 = 104$
$= 4 \cdot 26$

$x = \dfrac{-4 \pm 2\sqrt{26}}{-2}$

$= 2 \pm \sqrt{26}$

$ED(f) = \left]2-\sqrt{26} ; 2+\sqrt{26}\right[$

$f(x) \geq 0 \iff -x^2 + 4x + 22 \geq 1 \quad | -1$

$-x^2 + 4x + 21 \geq 0 \quad | \cdot(-1)$ ⚠

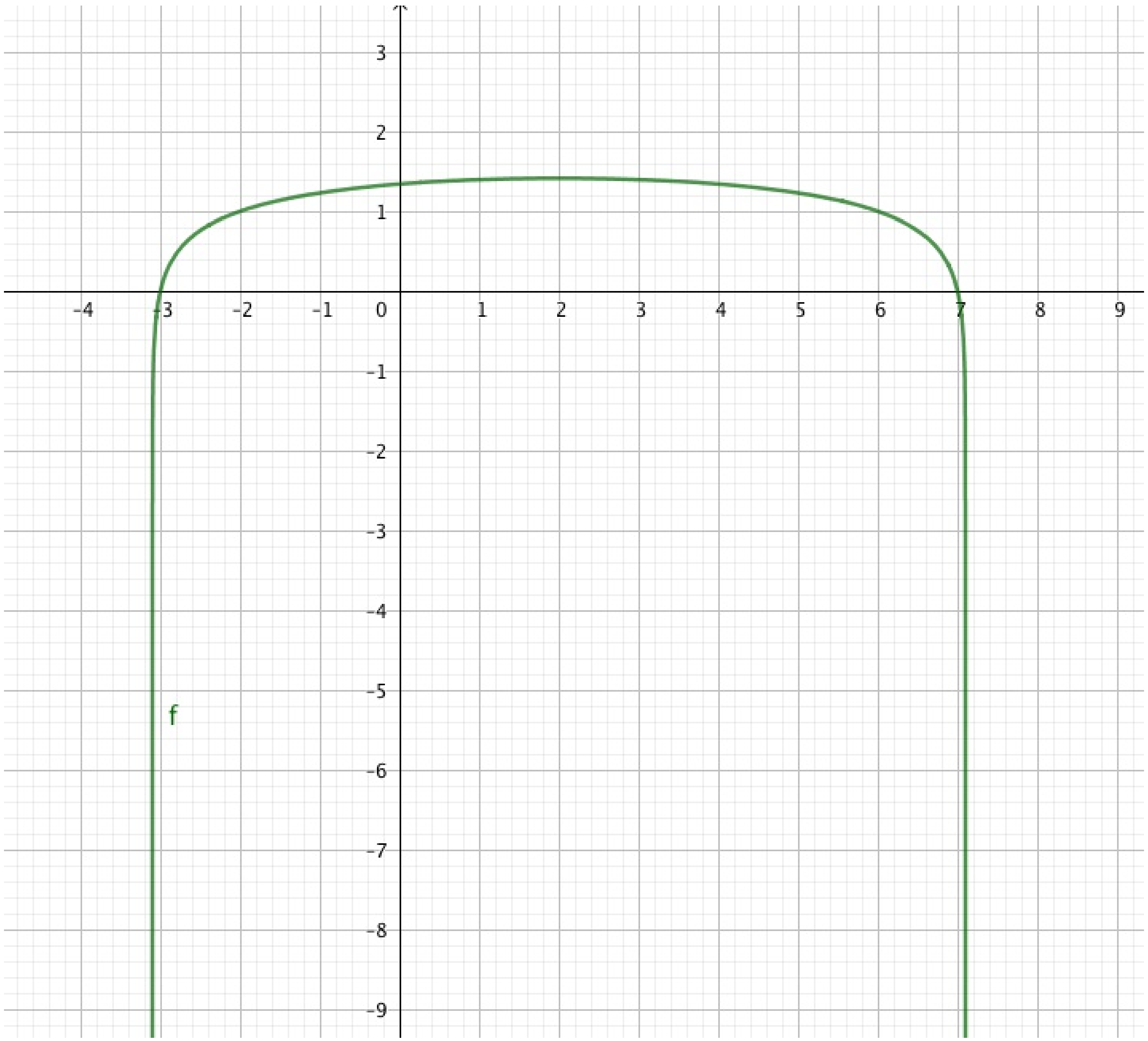
$x^2 - 4x - 21 \leq 0$

$(x-7)(x+3) \leq 0$

x		-3		7	
$x^2-4x-21$	$+$	0	$-$	0	$+$

Tableau des signes de $f(x)$

x		$2-\sqrt{26}$		-3		7		$2+\sqrt{26}$	
$f(x)$		‖	$-$	0	$+$	0	$-$	‖	



3
2
1
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
-1
-2
-3
-4
-5
-6
-7
-8
-9
f

$\log(X)$

① ED $X > 0$

② $0 < x < 1$ $\quad$ $X = 1$ $\quad$ $X > 1$

$-$ $\quad$ 0 $\quad$ $+$

$f(x) = \log(X - 1)$

		1		2	
$f(x)$		$\|$	$-$	0	$+$

b) $f(x) = 12 - 10^{3-x}$

$ED(f) = \mathbb{R}$

Zéro de $f(x)$: $\quad 12 = 10^{3-x}$

$$3 - x = \log(12)$$

$$x = -\log(12) + 3$$

$$f(3 - \log(12)) = 12 - 10^{3-(3-\log(12))}$$

$$= 12 - 10^{\log(12)} = 12 - 12 = 0$$

x		$3 - \log(12)$	
$f(x)$	$-$	0	$+$

$$f(-7) = 12 - 10^{3-(-7)} = 12 - 10^{10}$$