

16.01.20

$$d) f(x) = \frac{2x - \sqrt{4x^2 + 2x - 5}}{x + 3}$$

$$\lim_{x \rightarrow -\infty} \frac{2x - \sqrt{4x^2 + 2x - 5}}{x + 3} = \text{FI} \quad \frac{"-\infty"}{-\infty}$$

$$\lim_{x \rightarrow -\infty} \frac{2x - \sqrt{4x^2 + 2x - 5}}{x + 3} - \frac{2x + \sqrt{4x^2 + 2x - 5}}{2x + \sqrt{4x^2 + 2x - 5}} =$$

$$\lim_{x \rightarrow -\infty} \frac{4x^2 - (4x^2 + 2x - 5)}{(x+3)(2x + \sqrt{4x^2 + 2x - 5})} =$$

$$\lim_{x \rightarrow -\infty} \frac{-2x + 5}{(x+3)(2x + \sqrt{4x^2 + 2x - 5})} =$$

$$\lim_{x \rightarrow -\infty} \frac{-2x + 5}{(x+3)(2x + \sqrt{4x^2 (1 + \frac{2}{x} - \frac{5}{4x^2})})} =$$

$$\lim_{x \rightarrow -\infty} \frac{-2x + 5}{(x+3)(2x + |2x| \sqrt{1 + \dots})} = *$$

$$\lim_{x \rightarrow -\infty} \frac{-2x + 5}{(x+3)(2x - 2x \cdot \sqrt{1 + \dots})}$$

$$d) \ f(x) = \frac{2x - \sqrt{4x^2 + 2x - 5}}{x+3}$$

$$\textcircled{1} \quad \lim_{x \rightarrow -\infty} \frac{2x - |2x| \sqrt{1+\dots}}{x(1+\frac{3}{x})} = \lim_{x \rightarrow -\infty} \frac{2x + 2x \sqrt{1+\dots}}{x(1+\frac{3}{x})}$$

$$\lim_{x \rightarrow -\infty} \frac{2x(1 + \sqrt{1+\dots})}{x(1+\frac{3}{x})} = 4$$

$$\textcircled{2} \quad \star = \lim_{x \rightarrow +\infty} \frac{-2x+5}{(x+3)(2x+2x\sqrt{1+\dots})}$$

$$= \lim_{x \rightarrow +\infty} \frac{x(-2 + \frac{5}{x})}{(x+3) 2x(1 + \sqrt{1+\dots})} = \bigcirc$$

$$e) f(x) = 2x - \cos(x)$$

$$f) f(x) = \frac{2x - \cos(x)}{x - 1}$$

$$e) \lim_{x \rightarrow -\infty} (2x - \cos(x)) = -\infty$$

$$\lim_{x \rightarrow +\infty} (2x - \cos(x)) = +\infty$$

$$f) \lim_{x \rightarrow \infty} \left(\frac{2x}{x-1} - \frac{\cos(x)}{x-1} \right) = 2$$

$$\lim_{x \rightarrow \infty} \frac{2x}{x(1 - \frac{1}{x})}$$

$$g) \quad f(x) = x \sin\left(\frac{1}{x}\right)$$

$$h) \quad f(x) = \sin\left(\frac{1}{x}\right)$$

$$h) \quad \lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right) = 0$$

$$g) \quad \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{t \rightarrow 0} \frac{\sin(t)}{t} = 1$$

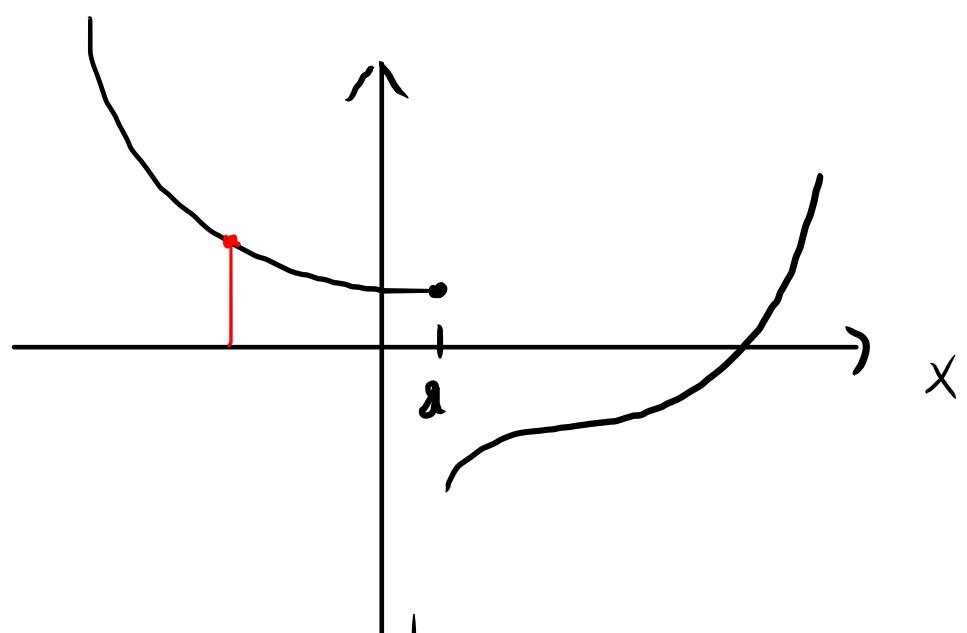
$$\frac{1}{x} = t$$

changement
de variable

Continuité

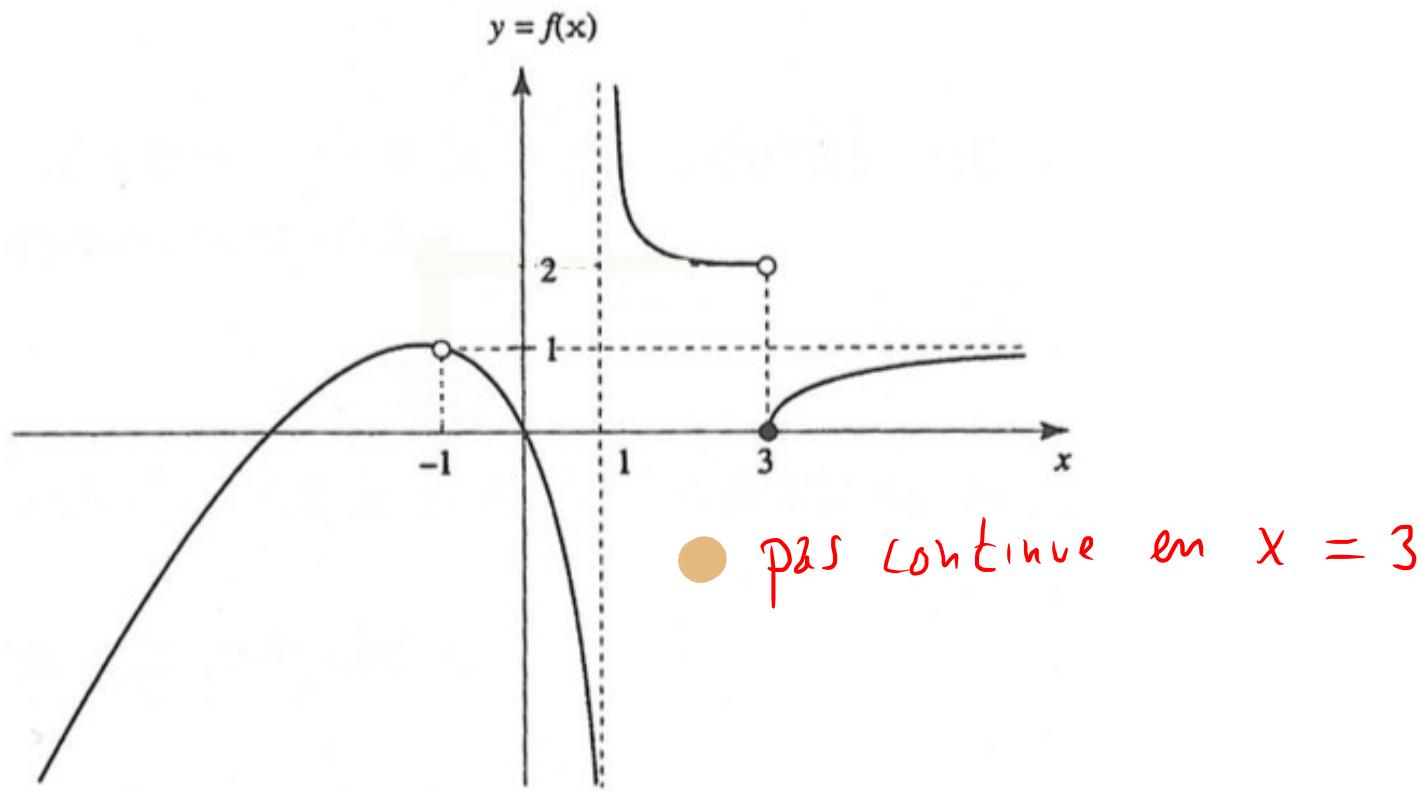
Une fonction f est continue en $x = a \in ED(f)$

Si $\lim_{x \rightarrow a} f(x) = f(a)$



pas continue en a

2.6.1 Déterminer la nature des discontinuités de la fonction f dont le graphe est représenté ci-dessous :



● pas continue en $x = 3$

$$ED(f) = \mathbb{R} - \{-1; 1\}$$

- $x = -1$ point trou
- $x = 1$ asymptote verticale

$$\lim_{x \rightarrow 3^-} f(x) = 2 ; \quad \lim_{x \rightarrow 3^+} f(x) = 0 = f(3)$$

2.6.2 Déterminer la nature des discontinuités des fonctions suivantes :

a) $f(x) = \frac{3}{x+2}$

b) $f(x) = \frac{x^2 + 4x + 3}{x^2 - 1}$

c) $f(x) = x^2 - 2x + 1$

d) $f(x) = \frac{x^3 + x^2 - 5x}{x^4 - 5x^3}$

e) $f(x) = \begin{cases} x-3 & , \text{ si } x \leq -1 \\ 4-2x & , \text{ si } x > -1 \end{cases}$

f) $f(x) = \frac{\sin(x)}{x}$

a) $ED(f) = \mathbb{R} - \{-2\}$ x = -2 AV

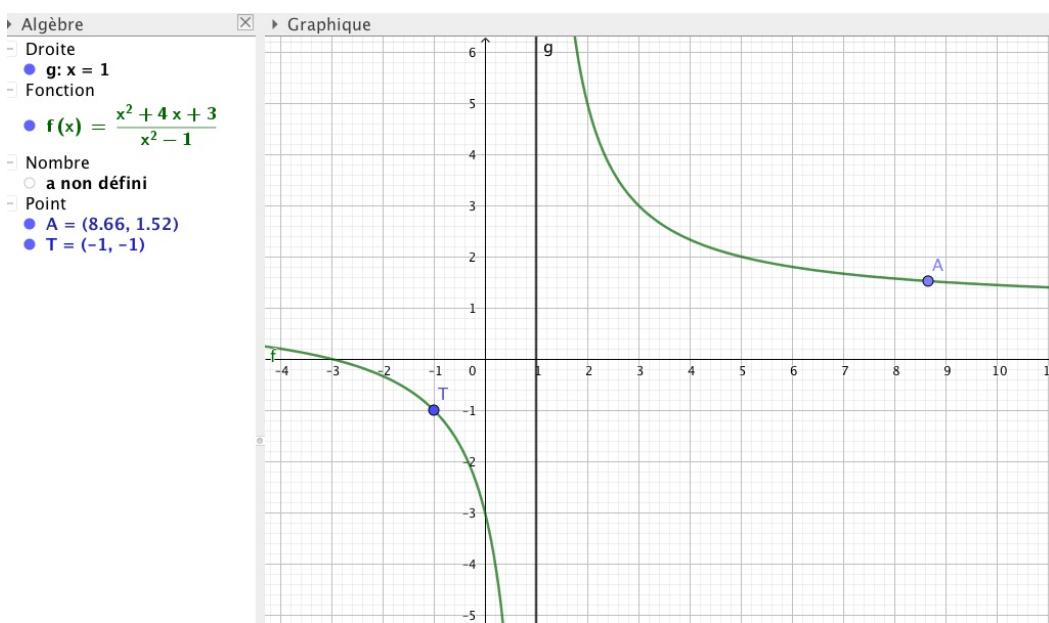
$EC(f) = ED(f)$

b) $ED(f) = \mathbb{R} - \{-1, 1\} = EC(f)$

$$\lim_{x \rightarrow -1} f(x) \stackrel{\text{FI}}{=} \lim_{x \rightarrow -1} \frac{(x+1)(x+3)}{(x+1)(x-1)} = \frac{2}{-2} = -1$$

Point trou T(-1, -1)

$$\lim_{x \rightarrow 1} f(x) = \infty \quad x = 1 \text{ est une AV}$$



$$e) \quad f(x) = \begin{cases} x - 3 & , \text{ si } x \leq -1 \\ 4 - 2x & , \text{ si } x > -1 \end{cases}$$

$$ED(f) = \mathbb{R}$$

$$\boxed{\lim_{x \rightarrow -1} f(x) = f(-1)}$$

$$f(-1) = -1 - 3 = -4$$

$$\lim_{\substack{x \rightarrow -1 \\ <}} f(x) = -4$$

$$\lim_{\substack{x \rightarrow -1 \\ >}} f(x) = 4 - (-2) = 6$$

Comme $\lim_{x \rightarrow -1} f(x)$ n'est pas définie, f est discontinue

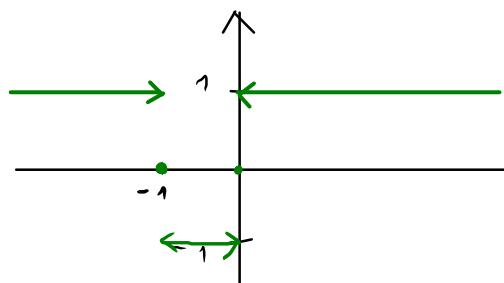
$$\text{en } x = -1$$

d) $f(x) = \operatorname{sgn}(x^2 + x)$

e) $f(x) = E(x) - x$

d)

x	$\chi(x+1)$				
	-				
x^2+x	+	0	-	0	+



$$\text{ED}(f) = \mathbb{R}$$

$$\text{EC}(f) = \mathbb{R} - \{-1, 0\}$$

$$f(x) = E(x) - x$$

$$\text{ED}(f) = \mathbb{R}$$

$$E(x) = \begin{cases} -1 & -1 \leq x < 0 \\ 0 & 0 \leq x < 1 \\ 1 & 1 \leq x < 2 \end{cases}$$

$$\text{EC}(f) = \mathbb{R} - \mathbb{Z}$$

$$f(0) = 0$$

$$\lim_{\substack{x \rightarrow 0 \\ >}} E(x) - x = 0$$

$$\lim_{\substack{x \rightarrow 0 \\ <}} E(x) - x = -1$$

