

Point d'inflexion

18.03.20

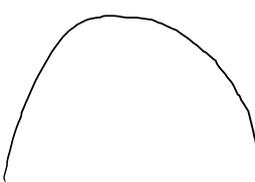
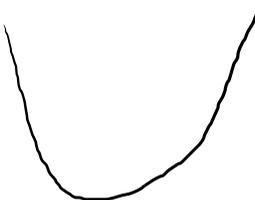
① $f(x) = x^3$
 $f'(x) = 3x^2$
 $f''(x) = 6x$

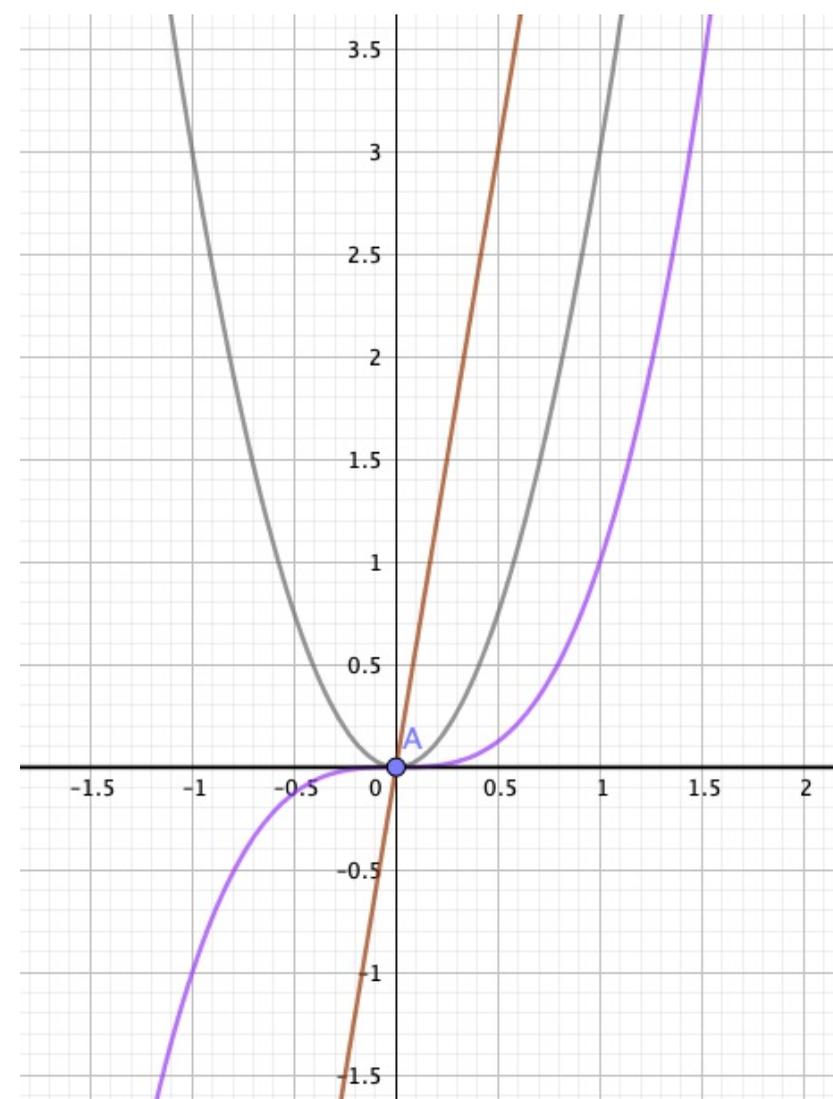
$ED(f) = \mathbb{R}$

$ED(f') = \mathbb{R}$

$ED(f'') = \mathbb{R}$

Tableau de la courbure

x	0	
$f''(x)$	-	+
$f(x)$	 concave	 convexe



$$\textcircled{2} f(x) = \frac{1}{10} (x^3 + x)$$

$$\text{ED}(f) = \mathbb{R}$$

$$f'(x) = \frac{1}{10} (3x^2 + 1)$$

$$\text{ED}(f') = \mathbb{R}$$

$$f''(x) = \frac{1}{10} \cdot 6x = \frac{3}{5}x$$

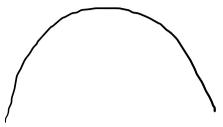
$$\text{ED}(f'') = \mathbb{R}$$

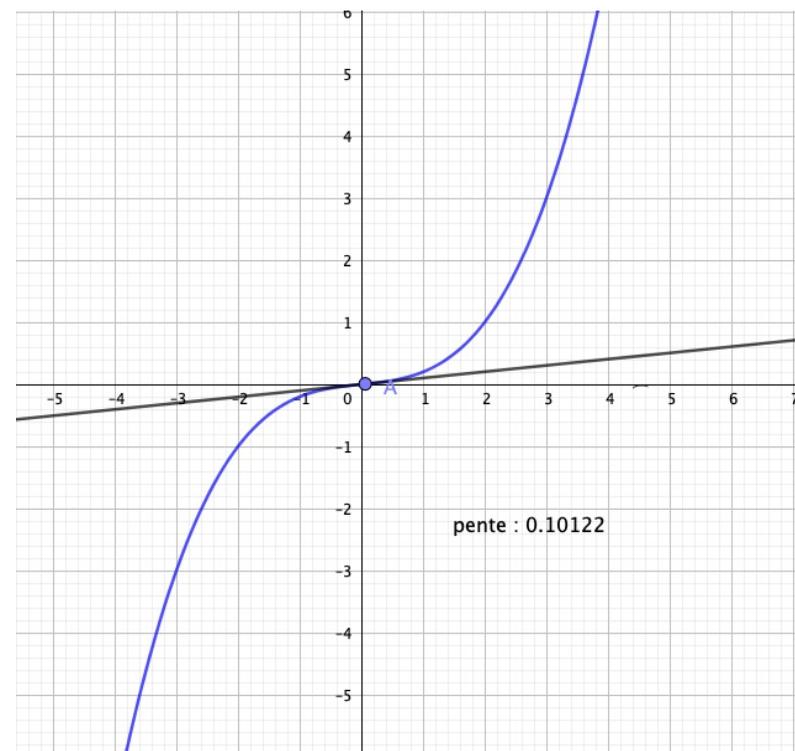
$$(k u)' = k \cdot u', \quad k \in \mathbb{R}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

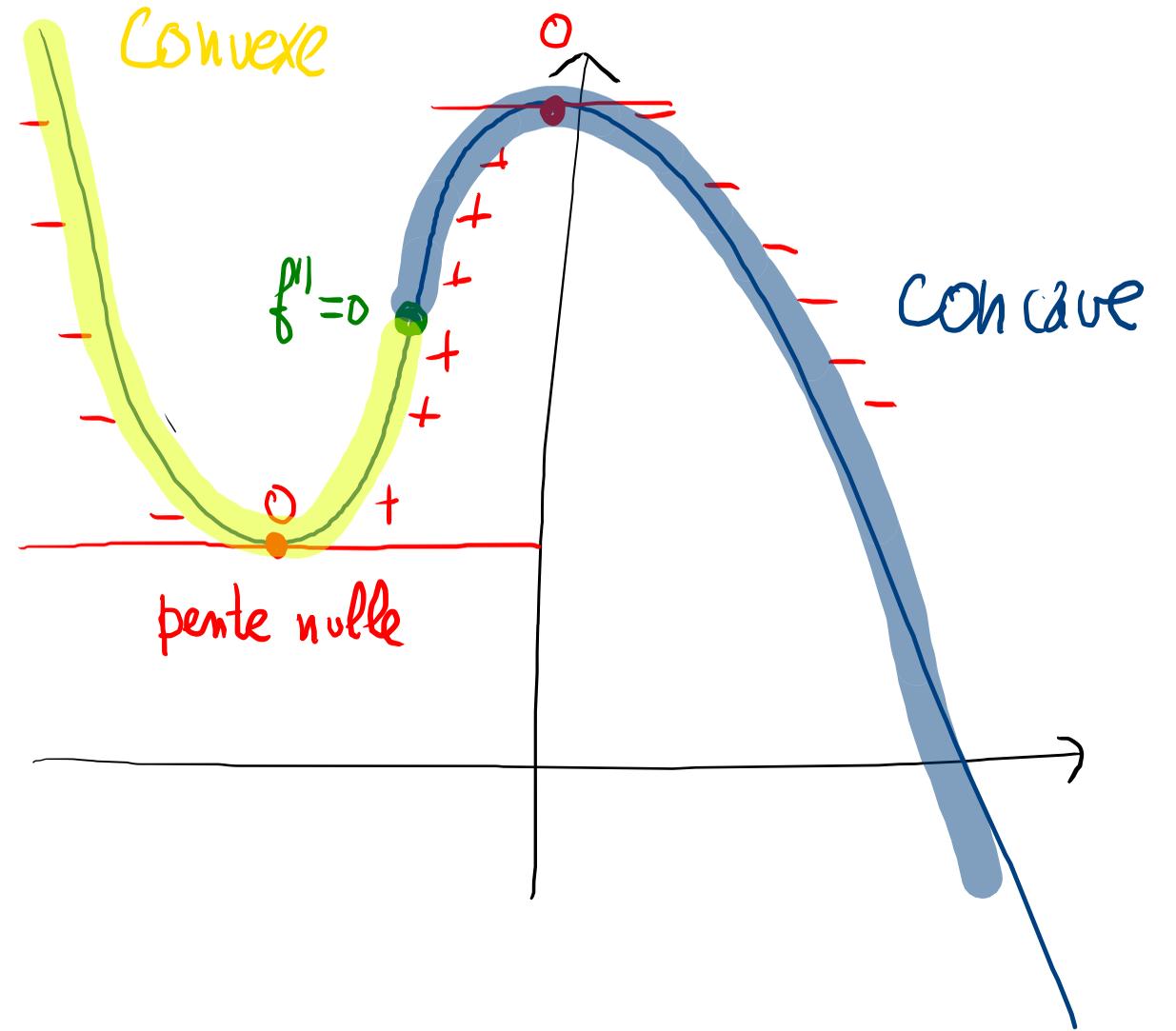
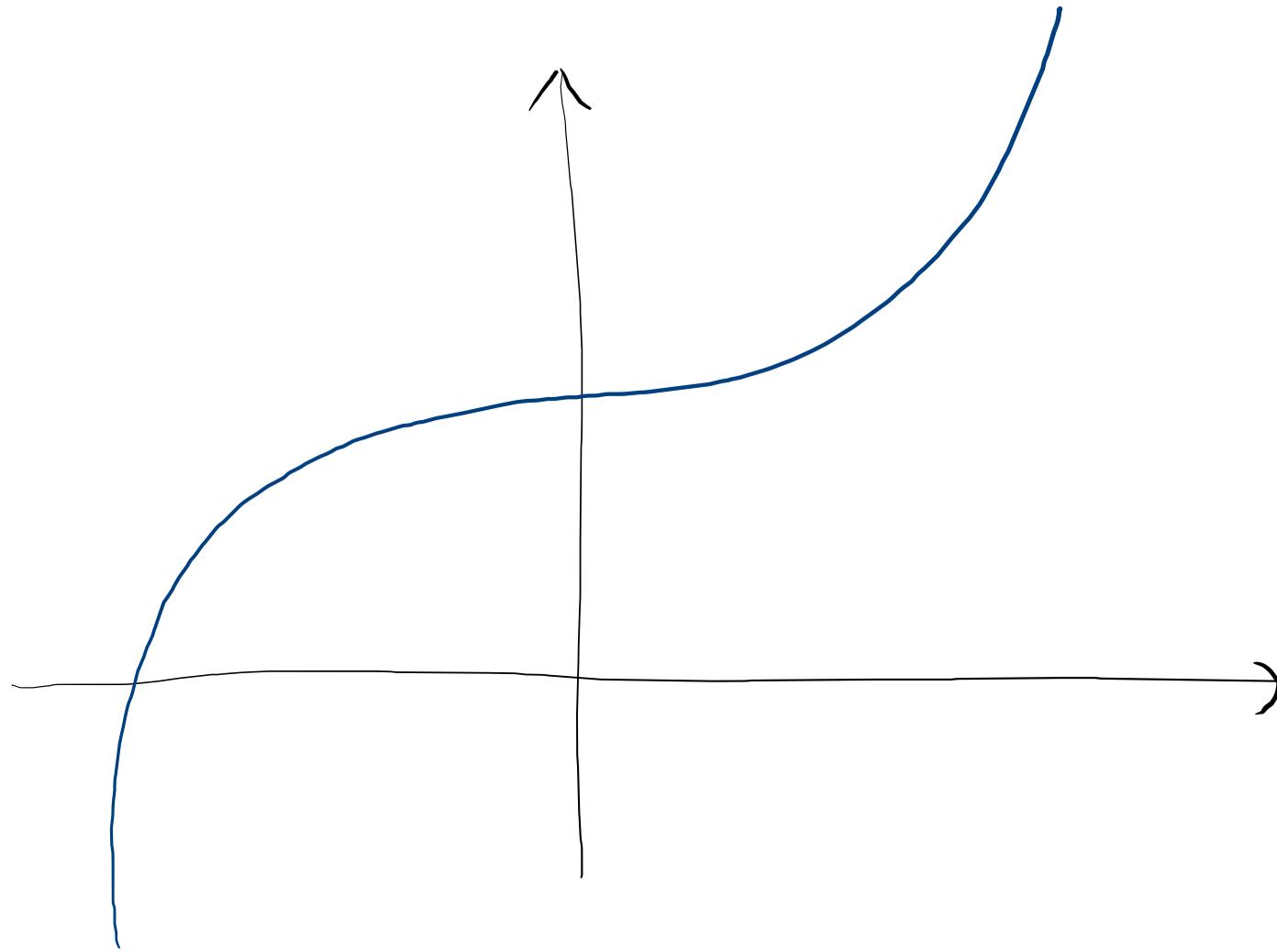
$$\left(\frac{x^3 + x}{10}\right)' = \frac{10^1 (3x^2 + 1) - 0}{10^2}$$

Tableau de la courbure

x	0		
$f''(x)$	-	0	+
$f(x)$		\boxed{pi}	



Le point d'inflexion (point où la courbe s'infléchit) est le point où la dérivée seconde s'annule et change de signe.



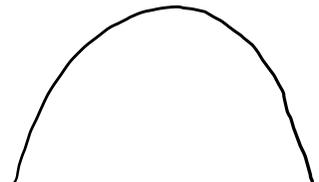
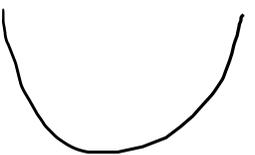
2.9.7

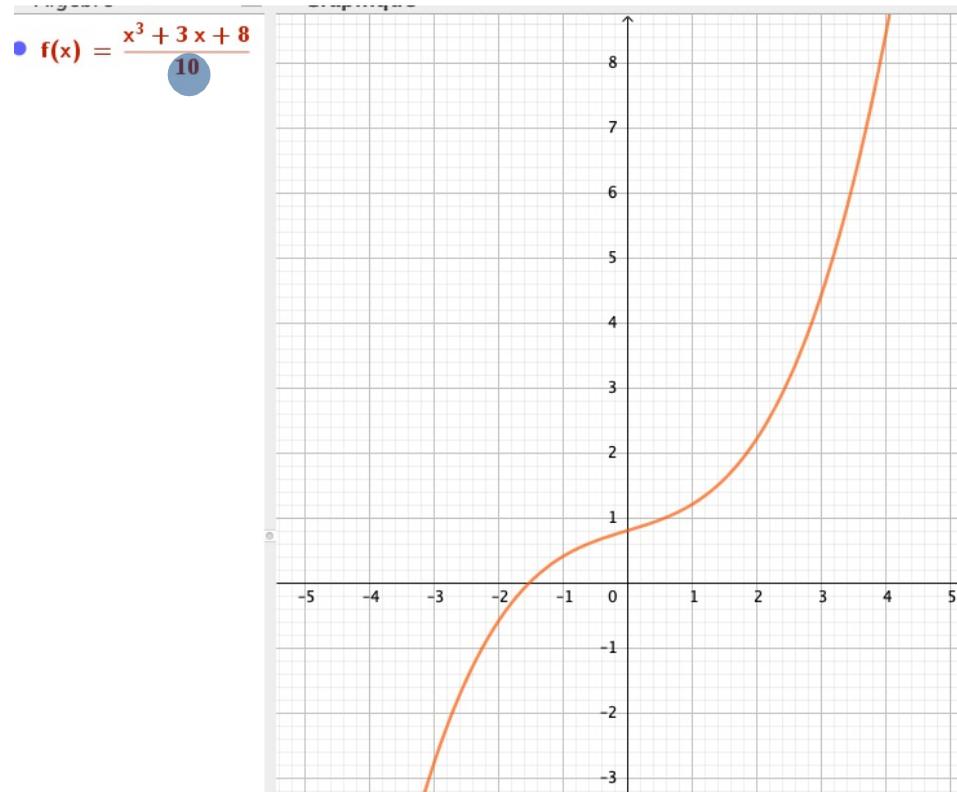
b) $f(x) = x^3 + 3x + 8$

$ED(f) = \mathbb{R}$

$f'(x) = 3x^2 + 3$

$f''(x) = 6x$

x	0		
$f''(x)$	-	0	+
$f(x)$		<div style="border: 1px solid black; padding: 2px; display: inline-block;">pi</div>	



pi (0; 8)

pi : point d'inflexion

d) $f(x) = \frac{1}{x^2 + 1}$

$ED(f) = \mathbb{R}$

$\left(\frac{1}{u}\right)' = \frac{-u'}{u^2}$

$f'(x) = \frac{-2x}{(x^2 + 1)^2}$

$ED(f') = \mathbb{R}$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$f''(x) = \frac{[-2(x^2 + 1)^2] - [(-2x) \cdot 2(x^2 + 1)^1 \cdot 2x]}{(x^2 + 1)^4}$

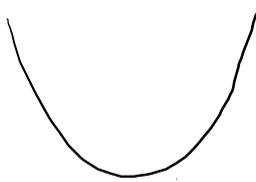
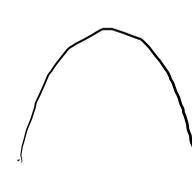
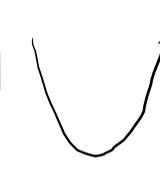
$= \frac{2(x^2 + 1) [- (x^2 + 1) + 2x \cdot 2x]}{(x^2 + 1)^{4-3}} = \frac{2(3x^2 - 1)}{(x^2 + 1)^3}$

$ED(f'') = \mathbb{R}$

$f''(x) = 3x^2 - 1 = 0 \iff$

$x^2 - \frac{1}{3} = 0 \iff x = \pm \frac{1}{\sqrt{3}}$

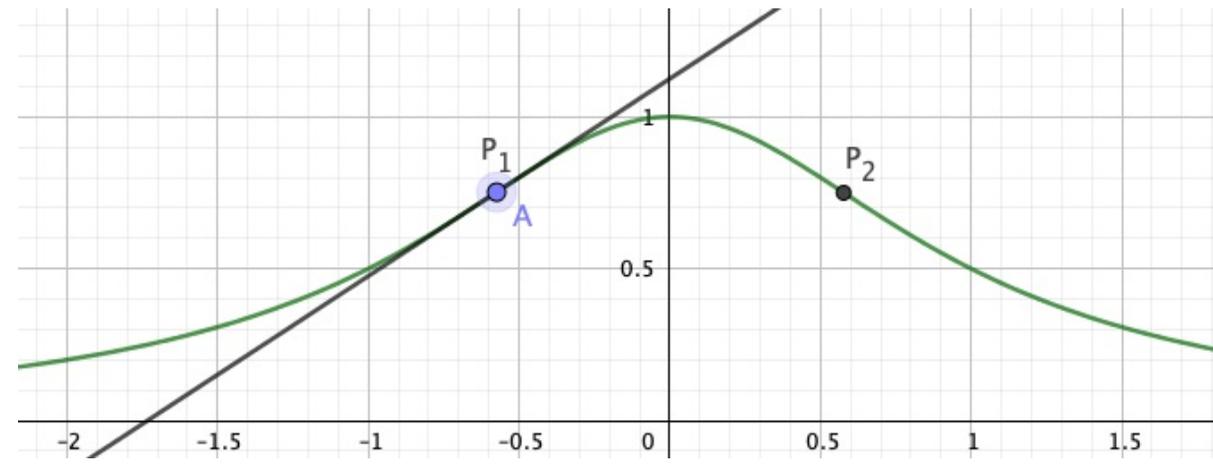
Tableau de la courbure

x	$-\frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{3}}$		
$f''(x)$	+	0	-	0	+
$f(x)$		pl		pl	

$pl \left(-\frac{1}{\sqrt{3}} ; \frac{3}{4} \right)$

$\frac{1}{\frac{1}{3} + 1} = \frac{3}{4}$

$pl \left(\frac{1}{\sqrt{3}} ; \frac{3}{4} \right)$



Eléonore

2.9.8 Déterminer l'équation de la tangente à la courbe $y = x^3 - 3x^2$ en son point d'inflexion.

2.9.8

$f'(x) = 3x^2 - 6x \rightarrow$ racine $3-6 = -3$

$f''(x) = 6x - 6$

x	1
Prv	- 0 +
$f(x)$	() () ()

$A(1, -2)$

$y = -3x + b$

$-2 = -3 + b$

$1 = b$

$\Rightarrow y = -3x + 1$

2.9.9

2.9.9 Déterminer les paramètres a , b et c tels que $f(x) = x^4 + ax^3 + bx^2 + c$ admette en $x = 1$ un point d'inflexion en lequel la tangente au graphe soit la droite d'équation $y = 16x - 5$.

$$f'(x) = 4x^3 + 3ax^2 + 2bx$$

$$f''(x) = 12x^2 + 6ax + 2b$$

$$\begin{cases} f''(1) = 0 \\ f'(1) = 16 \end{cases}$$

$$\begin{cases} 12 + 6a + 2b = 0 \\ 4 + 3a + 2b = 16 \end{cases}$$

$$\Leftrightarrow \begin{cases} 6a + 2b = -12 \\ 3a + 2b = 12 \end{cases} \begin{array}{c} b \\ 1 \end{array} \begin{array}{c} 0 \\ 1 \end{array}$$

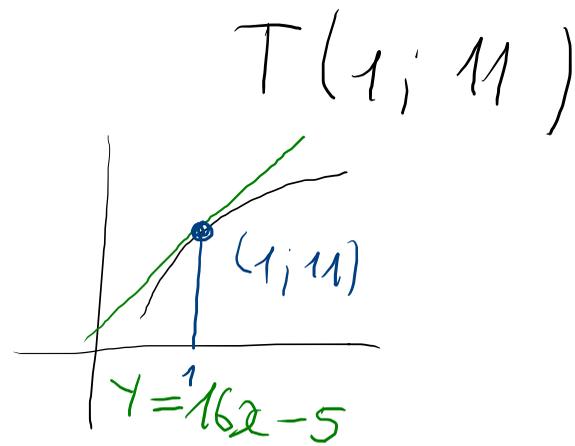
$$\cdot(-1) \quad \cdot(-2)$$

$$\begin{cases} 3a = -24 \\ -2b = -36 \end{cases} \Leftrightarrow \begin{cases} a = -8 \\ b = 18 \end{cases}$$

$$f(x) = x^4 - 8x^3 + 18x^2 + c$$

Comme $f(1) = 11$, on a : $1 - 8 + 18 + c = 11 \Rightarrow c = 0$

Ainsi : $f(x) = x^4 - 8x^3 + 18x^2$



2.9.10

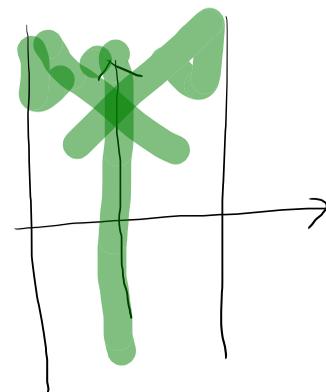
d) $f(x) = \frac{2x^2}{9-x^2}$

① Ensemble de définition : $9-x^2 = 0 \Leftrightarrow x = \pm 3$

$ED(f) = \mathbb{R} - \{-3, 3\}$

② Parité: $f(-x) = f(x) \Rightarrow f$ est paire sy

$f(-x) = \frac{2(-x)^2}{9-(-x)^2} = \frac{2x^2}{9-x^2} = f(x)$



③ Périodicité aucune

⑤ Asymptotes

5.1) AV: $\lim_{x \rightarrow -3} f(x) = \infty$
"18" / 0

AV: $x = -3$

$\lim_{x \rightarrow 3} f(x) = \infty$
"18" / 0

AV: $x = 3$

AH/AO: $\lim_{x \rightarrow \infty} \frac{2x^2}{-x^2+9} = \lim_{x \rightarrow \infty} \frac{2x^2}{-x^2} = -2$ AH: $y = -2$

④ Signe de f(x):

x	-3_s	0_d	3_s
$f(x)$	-	+ 0	+ -

⑥ Croissance

d) $f(x) = \frac{2x^2}{9-x^2}$

$$f'(x) = \frac{4x(9-x^2) - 2x^2 \cdot (-2x)}{(9-x^2)^2}$$

$$= \frac{4x [9-x^2 + x^2]}{(9-x^2)^2} = \frac{36x}{(9-x^2)^2}$$

$$ED(f') = ED(f)$$

Tableau de la croissance :

x	-3	0	3
$f'(x)$	-	0	+
$f(x)$	↘	min	↗

min (0; 0)

⑦ Courbure

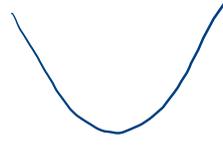
$$f''(x) = \frac{[36 (9-x^2)^2] - [36x \cdot 2(9-x^2)^1 \cdot (-2x)]}{(9-x^2)^4}$$

$$= \frac{36 \cancel{(9-x^2)} [(9-x^2) - x \cdot 2 \cdot (-2x)]}{(9-x^2)^{4-1}}$$

$$= \frac{36(9-x^2+4x^2)}{(9-x^2)^3} = \frac{36(3x^2+9)}{(9-x^2)^3} = \frac{108(x^2+3)}{(9-x^2)^3}$$

$$f'(x) = \frac{36x}{(9-x^2)^2}$$

Tableau de la courbure

x	-3	3	
$f''(x)$	-	+	-
$f(x)$			

Aucun pi !

⑧

Graphique de la fonction

