26.02.20

Dérivée d'une fonction composée

1) $(\sin(x))' = \cos(x)$

3) $\left(\left(\frac{2x}{3x+1}\right)^4\right)' =$

2) $(\sin(2x))' = 2 \cdot \cos(2x)$

4) $(\sin(x^2)) = \cos(x^2) \cdot 2x$

$f = g \circ u$

$x \xmapsto{\cdot 2} 2x \xmapsto{\sin} \sin(2x)$

$$f'(a) = \lim_{h \to 0} \frac{(g \circ u)(a+h) - (g \circ u)(a)}{h}$$

$$= \lim_{h \to 0} \frac{g(u(a+h)) - g(u(a))}{h}$$

$$\boxed{\text{posons } k = u(a+h) - u(a) \; ; \; \lim_{h \to 0} k = 0}$$

$$= \lim_{h \to 0} \frac{g(u(a) + k) - g(u(a))}{h} \cdot \frac{k}{k}$$

$$= \boxed{\lim_{h \to 0} \frac{g(u(a) + k) - g(u(a))}{k}} \cdot \boxed{\lim_{h \to 0} \frac{k}{h}}$$

$$= g'(u(a)) \times u'(a)$$

dérivée interne

$$\boxed{(g \circ u)' = (g(u))' = g'(u) \cdot u'}$$

$g(u(x))$

Exemple

$$f(x) = \left(\boxed{\frac{2x}{3x+1}}\right)^{3}$$

$$(x^3)' = 3x^2$$

$$f'(x) = 3\left(\frac{2x}{3x+1}\right)^2 \cdot \underbrace{\left(\frac{2x}{3x+1}\right)'}_{\frac{(2x)'\cdot(3x+1) - 2x\cdot(3x+1)'}{(3x+1)^2}} = 3\,\frac{4x^2\cdot 2}{(3x+1)^4}$$

$$= \frac{6x+2-6x}{(3x+1)^2} = \frac{2}{(3x+1)^2}$$

$$f'(x) = \frac{24x^2}{(3x+1)^4}$$

$$\sin\left(\boxed{\cos\left(x^2\right)}\right) = \cos\left(\cos\left(x^2\right)\right) \cdot \left(-\sin\left(x^2\right)\right) \cdot 2x$$

$$= -2x \sin\left(x^2\right) \cos\left(\cos\left(x^2\right)\right)$$

2.8.8 Calculer la dérivée de chacune des fonctions suivantes :

a) $f(x) = (x+1)(x-3)$

b) $f(x) = x(x^2+5)$

c) $f(x) = (7x^2-4x+3)(5-2x)$

d) $f(x) = (2x-1)(2-2x)(1+x)$

d) $(u \cdot v \cdot w)' = (uv)'w + uvw'$

$= (u'v + uv')w + uvw'$

$= u'vw + uv'w + uvw'$

$f'(x) = 2(2-2x)(1+x) + (-2)(2x-1)(1+x) + (2x-1)(2-2x)$

$\Downarrow$

h) $f(x) = \dfrac{x^3 - 10x^2}{1 - x}$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$u = x^3 - 10x^2$; $u' = 3x^2 - 20x$

$v = 1 - x$ $\quad v' = -1$

$$f'(x) = \frac{(3x^2 - 20x) \cdot (1 - x) - (x^3 - 10x^2)(-1)}{(1 - x)^2}$$

$$= \frac{3x^2 - 3x^3 - 20x + 20x^2 + x^3 - 10x^2}{(1 - x)^2}$$

$$= \frac{-2x^3 + 13x^2 - 20x}{(1 - x)^2}$$

2.8.10 Calculer la dérivée de chacune des fonctions suivantes :

a) $f(x) = (2x+3)^4$ b) $f(x) = (3-x)^5$

a) $f'(x) = 4(2x+3)^3 \cdot (2x+3)'$

$= 8(2x+3)^3$

b) $f'(x) = -5(3-x)^4$

e) $f(x) = x^2(5x+2)^3$

$f'(x) = 2x(5x+2)^3 + x^2 \cdot 3(5x+2)^2 \cdot 5$

$= 2x(5x+2)^3 + 15x^2(5x+2)^2$

$= x(5x+2)^2\left[2(5x+2) + 15x\right]$

$= x \cdot (5x+2)^2 \cdot (25x+4)$