

30.01.20

b)  $f(x) = x + \sqrt{x^2 - 1}$

① Signe de  $x^2 - 1$  :

$x$	-1	1
$x^2 - 1$	+	-
	+	-

$ED(f) = ]-\infty; -1] \cup [1; +\infty[$

② Signe de  $f(x)$ : zéros :  $x = -\sqrt{x^2 - 1}$  |  $( )^2$    
 $x^2 = x^2 - 1$

pas de zéro

$x$	-1	1
$f(x)$	-	// //
		+

$f(-1) = -1$   
 $f(1) = 1$

3) AV: aucune

4) AH/AO à gauche

$\lim_{x \rightarrow -\infty} x + \sqrt{x^2 - 1} \stackrel{\text{FI}}{=} \lim_{x \rightarrow -\infty} \frac{(x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1})}{x - \sqrt{x^2 - 1}}$   
 "-∞ + ∞"

$= \lim_{x \rightarrow -\infty} \frac{x^2 - (x^2 - 1)}{x - \sqrt{x^2(1 - \frac{1}{x^2})}} = \lim_{x \rightarrow -\infty} \frac{1}{x - |x| \sqrt{1 - \frac{1}{x^2}}} =$

$= \lim_{x \rightarrow -\infty} \frac{1}{x + x \sqrt{1 - \frac{1}{x^2}}} \stackrel{\frac{1}{\infty}}{=} 0 \Rightarrow \text{AHG } y = 0$

$|x| = -x$   
 $x < 0$

Pas intersection entre l'AHG et la courbe.

5) AH/AO à droite

$$\lim_{x \rightarrow +\infty} x + \sqrt{x^2 - 1} = +\infty + \infty = +\infty$$

pas d'AH à droite

$$m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{x + \sqrt{x^2 - 1}}{x}$$

$$= \lim_{x \rightarrow +\infty} \frac{x + \sqrt{x^2(1 - \frac{1}{x^2})}}{x} = \lim_{x \rightarrow +\infty} \frac{x + |x| \sqrt{1 - \frac{1}{x^2}}}{x}$$

$$= \lim_{\substack{|x|=x \\ x>0}} \frac{\cancel{x} (1 + \sqrt{1 - \frac{1}{x^2}})}{\cancel{x}} = 2$$

$$h = \lim_{x \rightarrow +\infty} (f(x) - mx)$$

$$= \lim_{x \rightarrow +\infty} (x + \sqrt{x^2 - 1} - 2x) = \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 1} - x)$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 - 1} - x)(\sqrt{x^2 - 1} + x)}{\sqrt{x^2 - 1} + x}$$

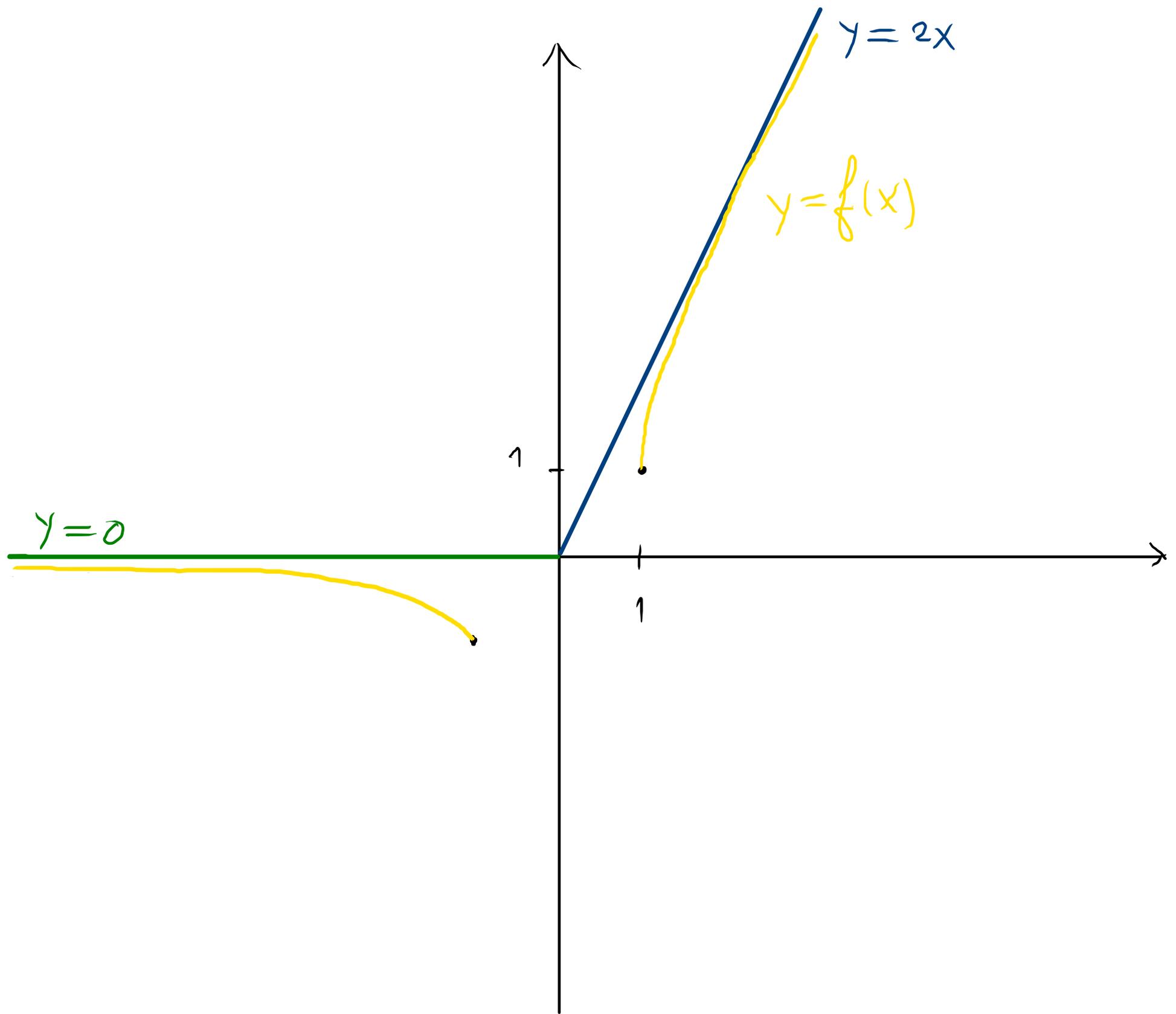
$$= \lim_{x \rightarrow +\infty} \frac{x^2 - 1 - x^2}{\sqrt{x^2 - 1} + x} \stackrel{\frac{-1}{\infty}}{=} 0$$

$$\text{AOD } y = 2x$$

Intersection entre l'AOD et la courbe  $y = 2x$

$$\begin{array}{l} x + \sqrt{x^2 - 1} = 2x \\ \sqrt{x^2 - 1} = x \\ x^2 - 1 = x^2 \end{array} \quad \left| \begin{array}{l} -x \\ (-) \end{array} \right. \quad \triangle$$

aucune intersection



## Exercice supplémentaire

$$1) f(x) = \frac{x^3 - 2x + 1}{2x^2 + 1}$$

$$2) f(x) = \sqrt{9x^2 + 6x} - 3x - 3$$

$$f(x) = \frac{x^3 - 2x + 1}{2x^2 + 1}$$

1)  $ED(f) = \mathbb{R}, \quad 2x^2 + 1 > 0$

2)  $p = x^3 - 2x + 1$

$p(1) = 1 - 2 + 1 = 0 \Rightarrow x-1 / p$

par Horner :

1	0	-2	1
1	1	1	-1
1	1	-1	0

$$f(x) = \frac{(x^2 + x - 1)(x - 1)}{2x^2 + 1}$$

$$x^2 + x - 1 = 0$$

$$\Delta = 5$$

$$x = \frac{-1 \pm \sqrt{5}}{2} \approx \begin{cases} -1,62 \\ 0,62 \end{cases}$$

x	$-\frac{1-\sqrt{5}}{2}$	0	$\frac{1+\sqrt{5}}{2}$
f(x)	- 0	+ 0	- 0 +

### 3) AO par division euclidienne

$$\begin{array}{r|l}
 X^3 & -2X + 1 \\
 - X^3 & + \frac{1}{2}X \\
 \hline
 & -\frac{5}{2}X + 1 \\
 & 2X^2 + 1 \\
 & \frac{1}{2}X
 \end{array}$$

$$f(x) = \frac{1}{2}x + \boxed{\frac{-\frac{5}{2}x + 1}{2x^2 + 1}} S(x)$$

$$\Rightarrow AO : y = \frac{1}{2}x$$

Intersection AO et la courbe :

$$-\frac{5}{2}x + 1 = 0$$

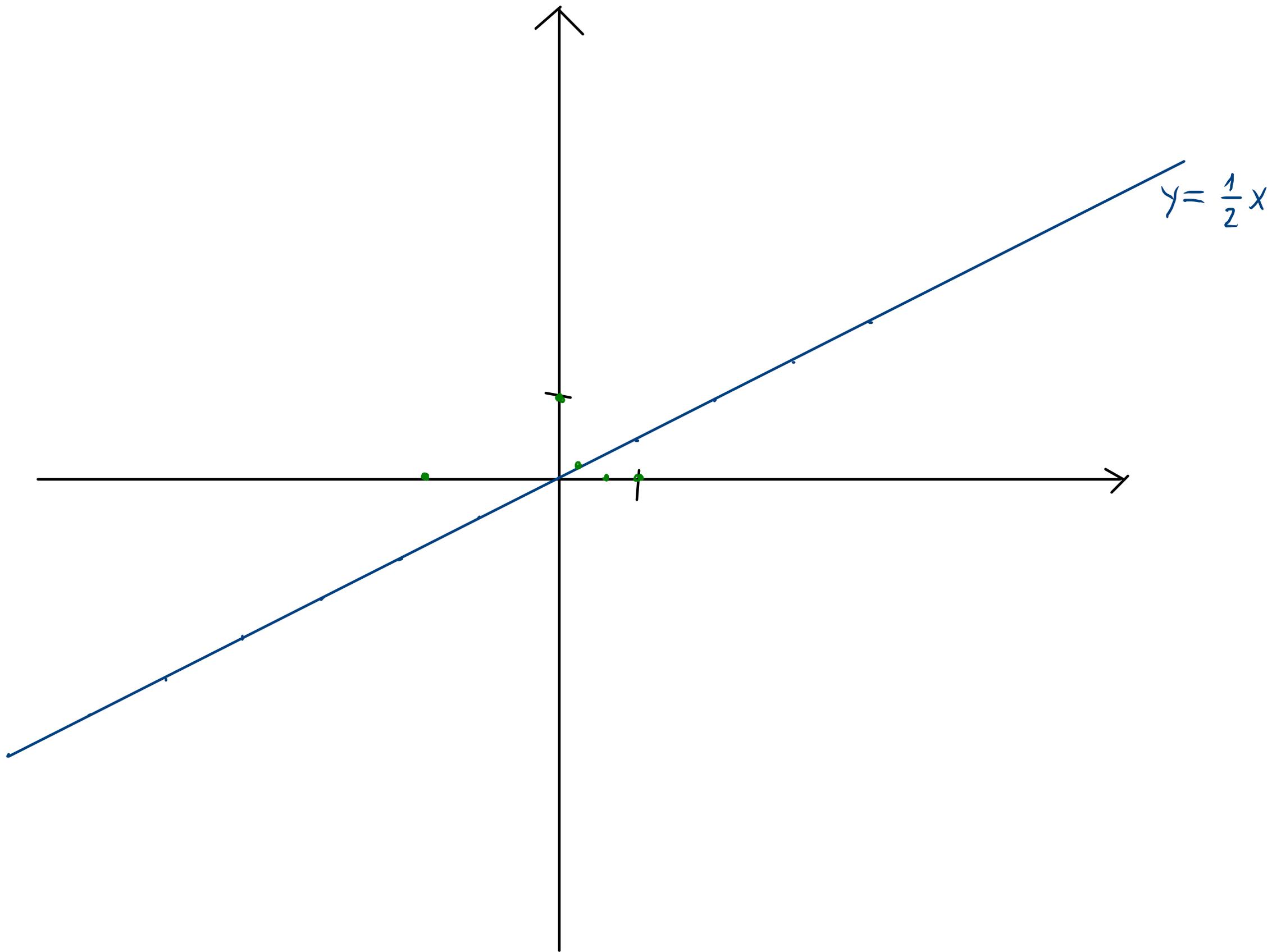
$$-\frac{5}{2}x = -1$$

$$x = \frac{2}{5}$$

Position entre l'AO et la courbe

x	$\frac{2}{5}$		
S(x)	+	0	-

au-dessus      au-dessous





Algèbre

Graphique

Droite

$$g: y = 0.5x$$

Fonction

$$f(x) = \frac{x^3 - 2x + 1}{2x^2 + 1}$$

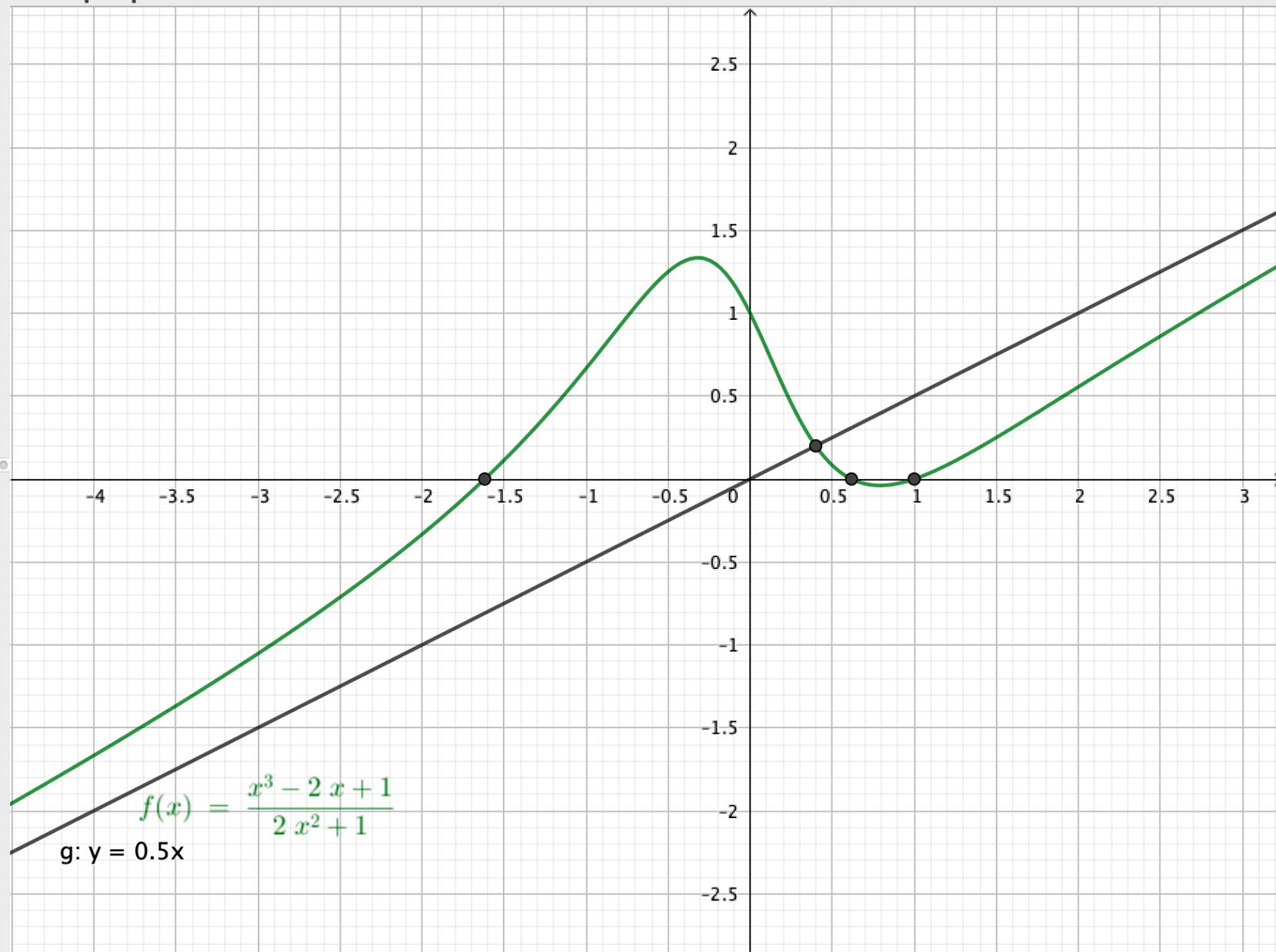
Point

$$A = (-1.618, 0)$$

$$B = (0.618, 0)$$

$$C = (1, 0)$$

$$D = (0.4, 0.2)$$



Saisie:



$$f(x) = \sqrt{9x^2 + 6x} - 3x - 3$$

1)  $9x^2 + 6x = 0$   
 $9x(x + \frac{2}{3}) = 0$

$x$	$-\frac{2}{3}$	$0$
$9x^2 + 6x$	$+$ $0$	$-$ $0$ $+$

$$ED(f) = ]-\infty; -\frac{2}{3}] \cup [0; +\infty[$$

2) zéro de  $f(x)$  :  $\sqrt{9x^2 + 6x} - 3x - 3 = 0$  |  $( )^2$  

$$\sqrt{9x^2 + 6x} = 3x + 3$$

$$9x^2 + 6x = 9x^2 + 18x + 9$$

$$-12x = 9$$

$$x = -\frac{9}{12} = -\frac{3}{4}$$

$x$	$-\frac{3}{4}$	$-\frac{2}{3}$	$0$
$f(x)$	$+$ $0$	$-$	$/$

$$f(-\frac{2}{3}) = -1$$

$$f(0) = -3$$

3) Avance AV cf ED(f)

4) AA/AD à droite

$$\lim_{x \rightarrow -\infty} (\sqrt{9x^2+6x} - (3x+3)) = +\infty - (-\infty) = +\infty + \infty = +\infty$$

pas d'ATG

$$\begin{aligned} M &= \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2+6x} - 3x - 3}{x} = \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2(1+\frac{2}{3x})} - 3x - 3}{x} \\ &= \lim_{x \rightarrow -\infty} \frac{|3x| \sqrt{1+\frac{2}{3x}} - 3x - 3}{x} = \lim_{x \rightarrow -\infty} \frac{-3x \sqrt{1+\frac{2}{3x}} - 3x - 3}{x} \\ &= \lim_{x \rightarrow -\infty} \frac{-3x \left( \sqrt{1+\frac{2}{3x}} + 1 + \frac{1}{x} \right)}{x} = -3 \cdot (1+1) = -6 \end{aligned}$$

$$R = \lim_{x \rightarrow -\infty} (\sqrt{9x^2+6x} - 3x - 3 - (-6x)) = \lim_{x \rightarrow +\infty} \sqrt{9x^2+6x} + 3x - 3$$

$$\textcircled{FI} \lim_{x \rightarrow -\infty} \frac{(\sqrt{9x^2+6x} + (3x-3))(\sqrt{9x^2+6x} - (3x-3))}{\sqrt{9x^2+6x} - (3x-3)}$$

$$= \lim_{x \rightarrow -\infty} \frac{9x^2+6x - (9x^2-18x+9)}{\sqrt{9x^2(1+\frac{2}{3x})} - 3x+3}$$

$$= \lim_{x \rightarrow -\infty} \frac{24x-9}{-3x \sqrt{1+\frac{2}{3x}} - 3x+3} = \lim_{x \rightarrow -\infty} \frac{24x \left( 1 - \frac{3}{8x} \right)}{-3x \left( \sqrt{1+\frac{2}{3x}} + 1 - \frac{1}{x} \right)}$$

$$= \frac{24}{-3 \cdot (1+1)} = \frac{24}{-6} = -4$$

$$\text{ADG: } y = -6x - 4$$

5) AH / AD  $\bar{a}$  dvoite

$$\lim_{x \rightarrow +\infty} \left( \sqrt{9x^2 + 6x} - (3x + 3) \right) \stackrel{FI}{=} +\infty - \infty$$

$$\lim_{x \rightarrow +\infty} \frac{\left( \sqrt{9x^2 + 6x} - (3x + 3) \right) \left( \sqrt{9x^2 + 6x} + (3x + 3) \right)}{\sqrt{9x^2 + 6x} + 3x + 3} =$$

$$\lim_{x \rightarrow +\infty} \frac{9x^2 + 6x - (9x^2 + 18x + 9)}{|3x| \sqrt{1 + \frac{2}{3x}} + 3x + 3} = \lim_{x \rightarrow +\infty} \frac{-12x - 9}{3x \left( \sqrt{1 + \frac{2}{3x}} + 1 + \frac{1}{x} \right)}$$

$$\begin{aligned} |3x| &= 3x \\ x &> 0 \end{aligned}$$

$$= \lim_{x \rightarrow +\infty} \frac{-12 \cancel{x} \left( 1 + \frac{3}{4x} \right)}{3 \cancel{x} \left( \sqrt{1 + \frac{2}{3x}} + 1 + \frac{1}{x} \right)} = \frac{-12}{3 \cdot (1+1)} = \frac{-12}{6} = -2$$

$$\text{AHD: } y = -2$$

Position entre les Att/H0 et la courbe  $y = f(x)$

à gauche :

$$\begin{aligned} S_g(x) &= \sqrt{9x^2 + 6x} - 3x - 3 - (-6x - 4) \\ &= \sqrt{9x^2 + 6x} + 3x + 1 \quad -\frac{2}{3} \leq x \leq 0 \end{aligned}$$

zéro de  $S_g(x)$  :  $\sqrt{9x^2 + 6x} = -(3x + 1) \quad | (\quad)^2 \quad \triangle$

$$9x^2 + 6x = 9x^2 + 6x + 1$$

aucune sol, donc aucune int.

$x$		$-\frac{2}{3}$	$0$
$S_g(x)$	+	///	+
	zu-dessous		zu-dessus

$$S_g(-1) = \sqrt{3} - 3 + 1 < 0$$

$$S_g(0) = 1$$

à droite :

$$\begin{aligned} S_d(x) &= \sqrt{9x^2 + 6x} - 3x - 3 - (-2) \\ &= \sqrt{9x^2 + 6x} - 3x - 1 \quad -\frac{2}{3} \leq x \leq 0 \end{aligned}$$

zéro de  $S_d(x)$  :  $\sqrt{9x^2 + 6x} = 3x + 1$

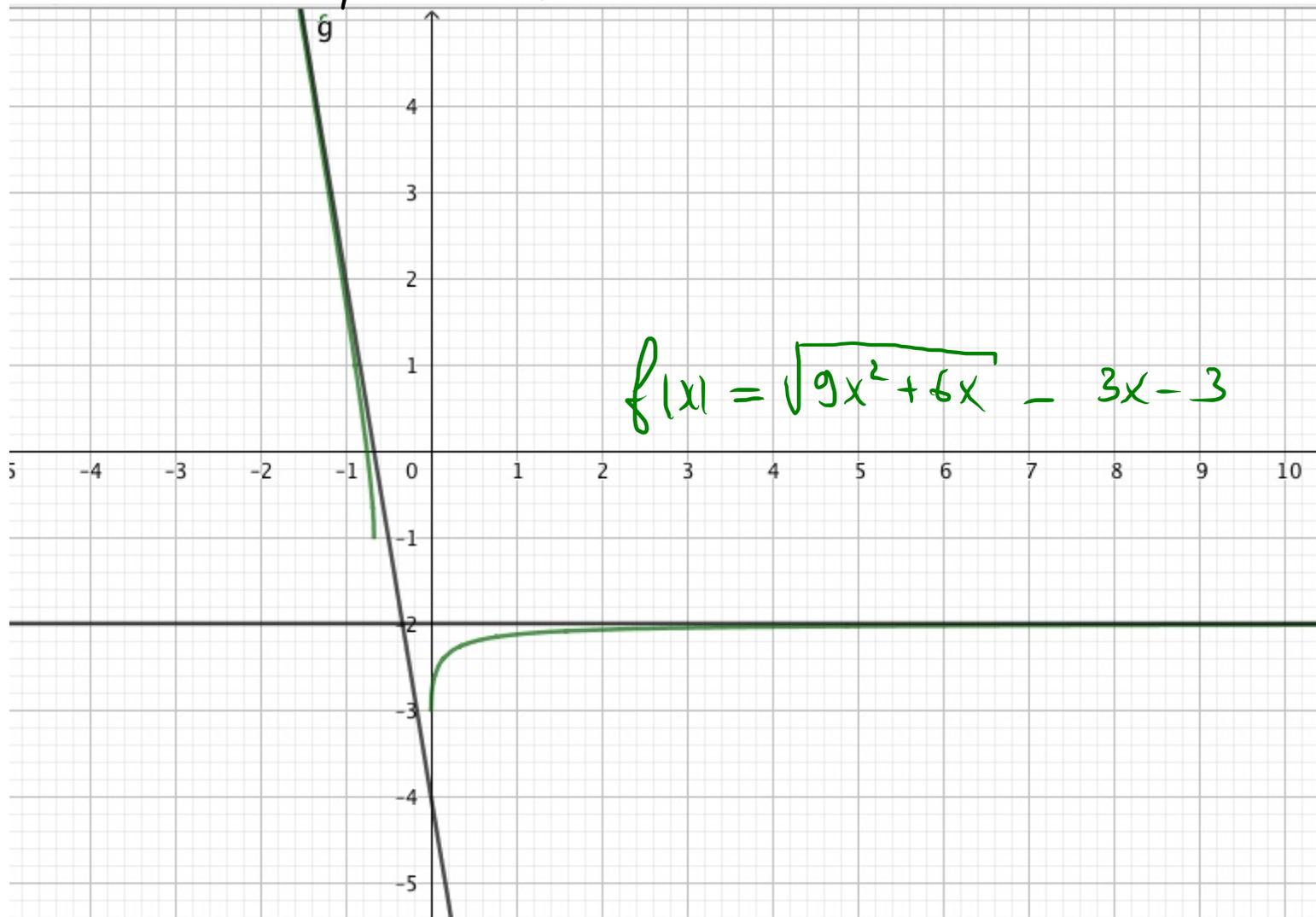
aucune sol, donc aucune intersection

$x$		$-\frac{2}{3}$	$0$
$S_d(x)$	+	///	-
	zu-dessus		zu-dessous

$$S_d(-1) = \sqrt{3} + 3 - 1$$

$$S_d(0) = -1$$

$$y = -6x - 4$$



$$y = -2$$