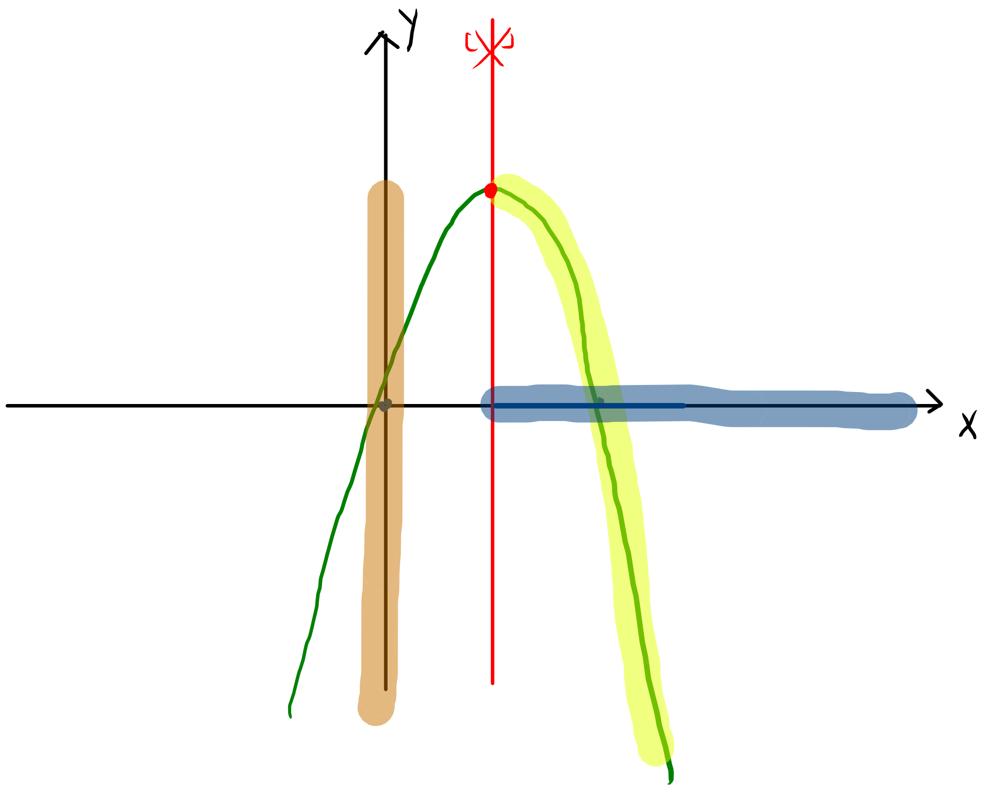
30.10.19

2.3.3 Déterminer l'ensemble de définition et l'ensemble d'arrivée pour que les fonctions suivantes soient des bijections. Puis donner leur réciproque.

c) $f(x) = -x^2 + 4x$ $= -x(x-4)$

$f(2) = -4 + 8 = 4$



$f : [2; +\infty[\longrightarrow]-\infty; 4]$

$x \longmapsto -x^2 + 4x$

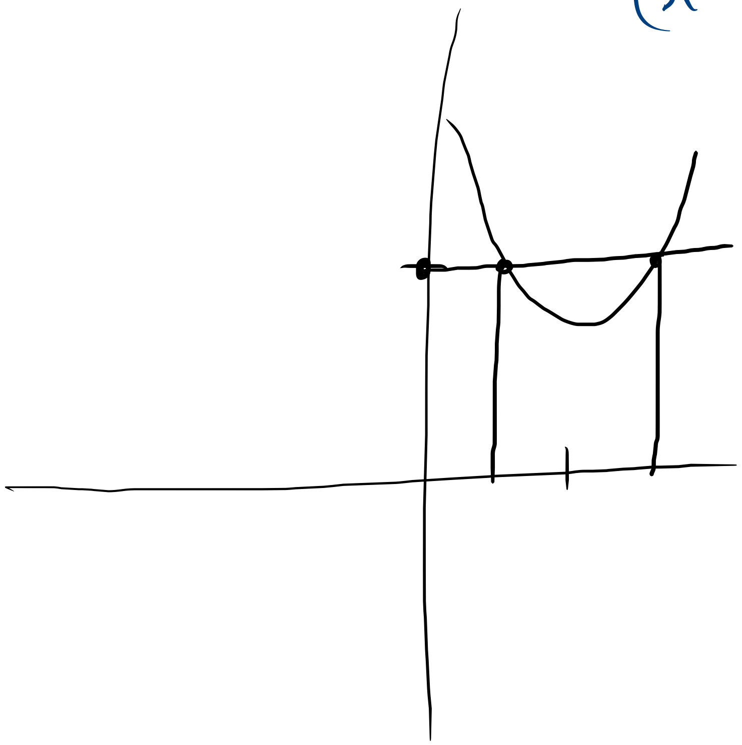
$f^r :]-\infty; 4] \longrightarrow [2; +\infty[$

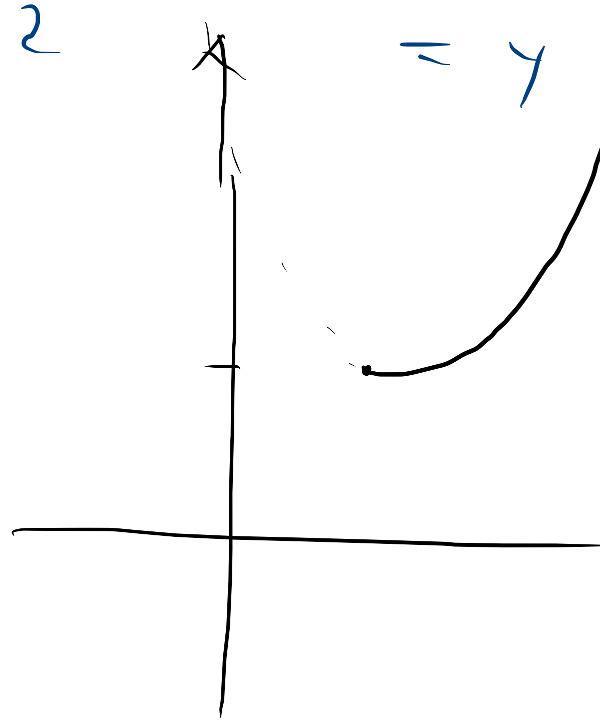
$x \longmapsto \sqrt{-x+4} + 2$

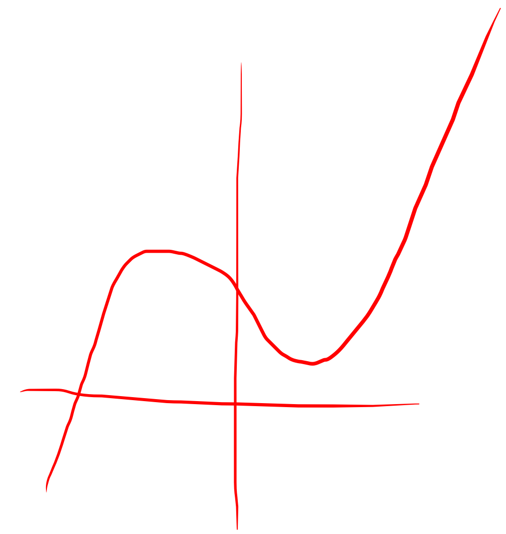
$$-x^2 + 4x = y$$
$$-(x^2 - 4x + 4) = y - 4$$
$$-(x-2)^2 = y - 4$$
$$(x-2)^2 = -y + 4 \qquad x \geq 2$$
$$x - 2 = \sqrt{-y+4} \qquad y \leq 4$$
$$x = 2 + \sqrt{-y+4}$$

$$x^2 - 6x + 9 = y + 9$$

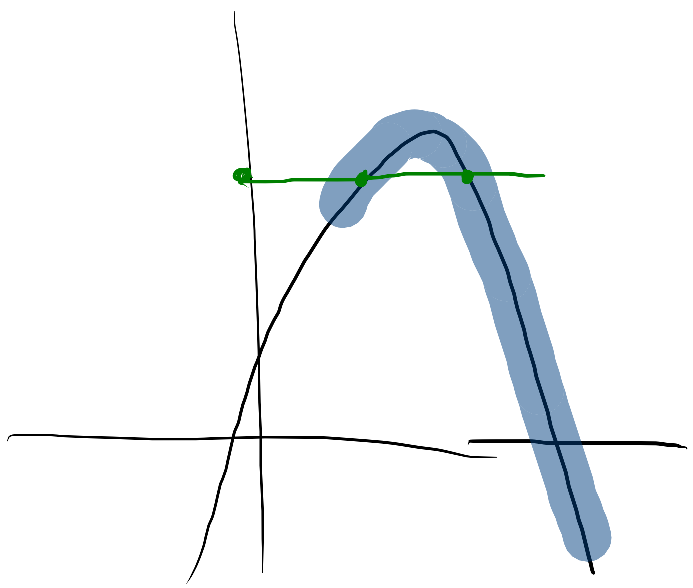
$$(x-3)^2 = y + 9$$

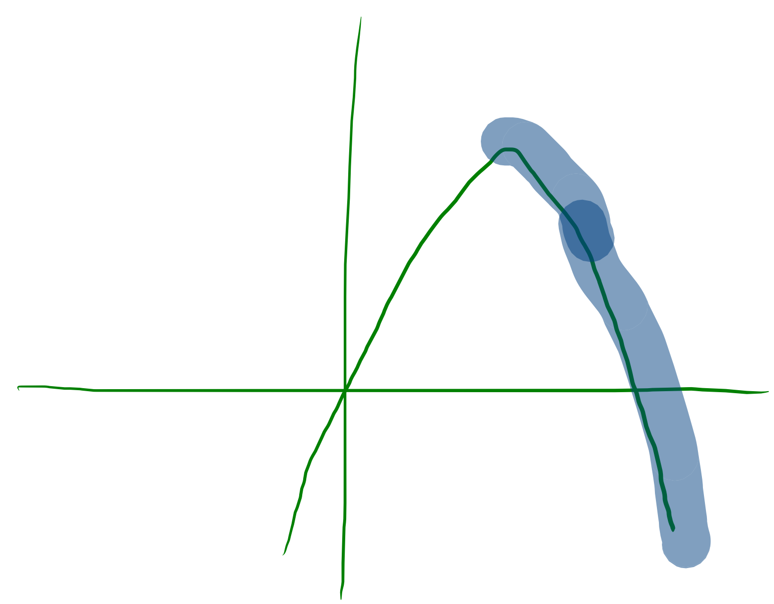




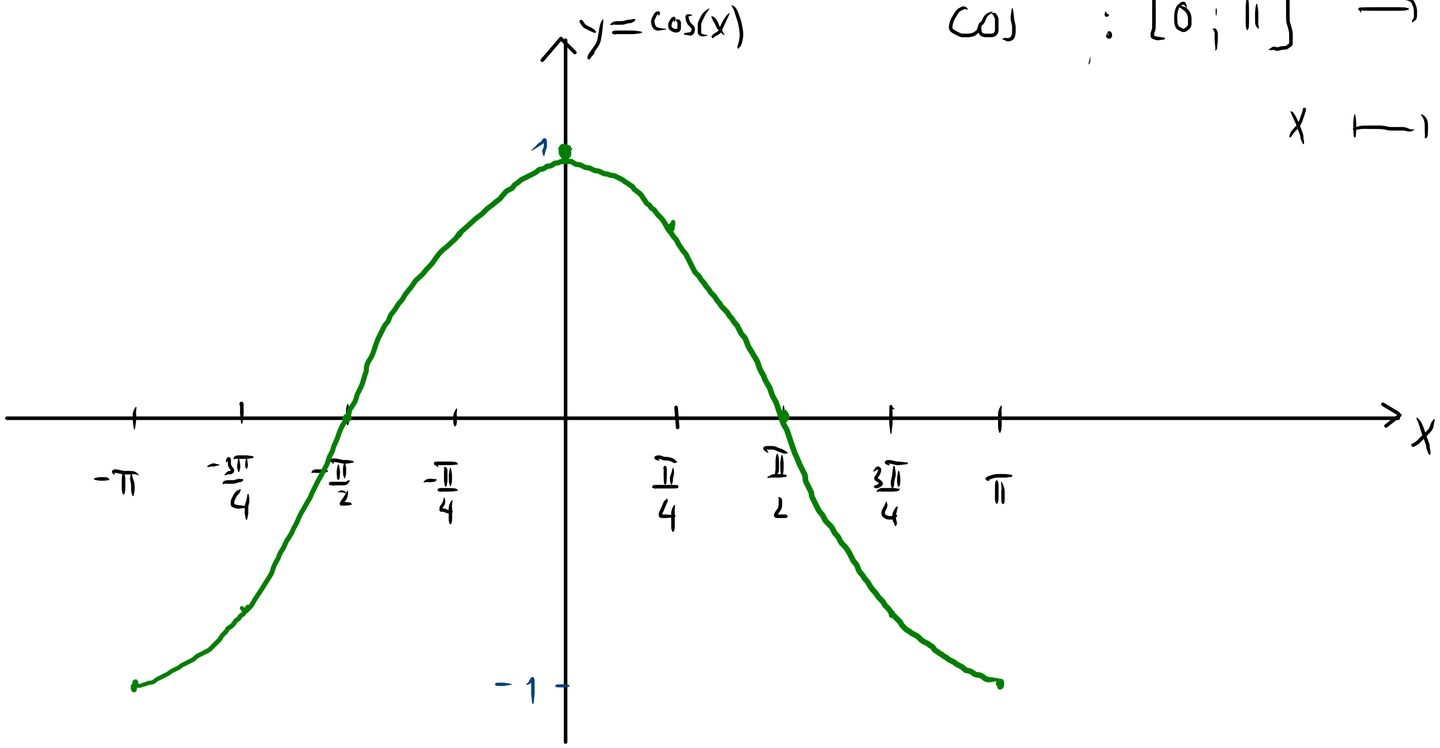


$$x^3 + 3x^2 - x$$



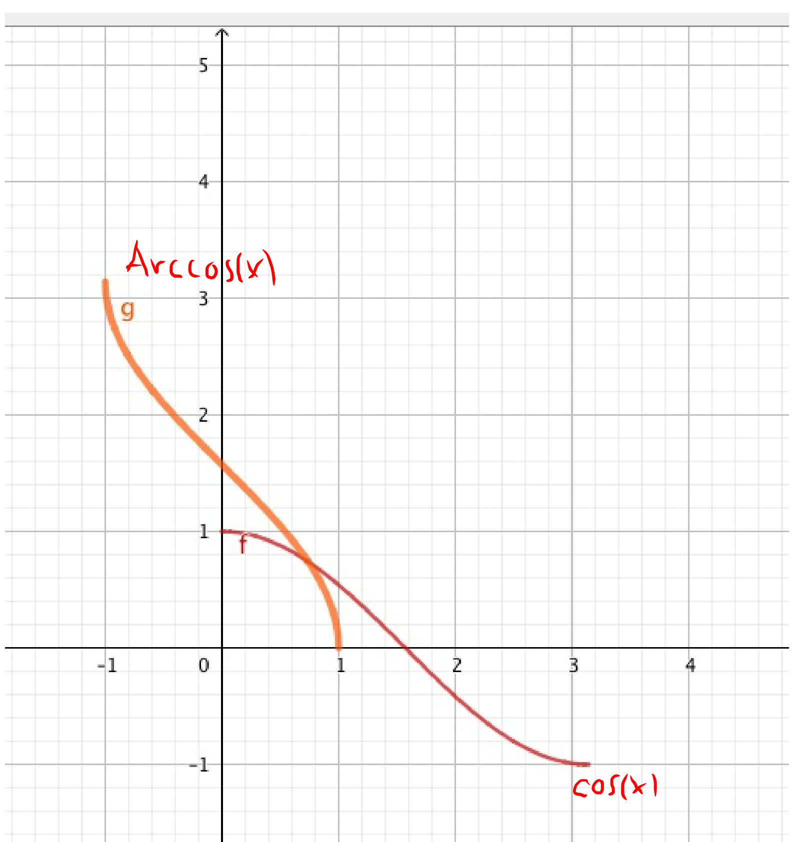


d) $f(x) = \cos(x)$



$\cos : [0 ; \pi] \to [-1, 1]$

$x \mapsto \cos(x)$



$\operatorname{Arccos} : [-1; 1] \to [0; \pi]$

$x \mapsto \operatorname{Arccos}(x)$

Chapitre 4

Puissances, racines, exponentielles et logarithmes

Soit $a \in \mathbb{R}_+^*$ et $n \in \mathbb{N}^*$. On définit

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ fois}}$$

a^n exposant

base

$\boxed{\begin{array}{l}\textbf{Règles}\end{array}}$

Règles

1. $$a^n \cdot a^m = a^{n+m}$$

2. $$(a^n)^m = a^{nm}$$

3. $$(a \cdot b)^n = a^n \cdot b^n$$

4. $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

5. $$\frac{a^n}{a^m} = \begin{cases} a^{n-m} & \text{si } n > m \\ 1 & \text{si } n = m \\ \frac{1}{a^{m-n}} & \text{si } n < m \end{cases}$$

Puissances à exposants entiers relatifs

6 $\quad a^0 \cdot a^m \underset{1}{=} a^{0+m} = a^m \Rightarrow a^0 = 1$

7 $\quad a^n \cdot a^{-n} \underset{1}{=} a^{n+(-n)} = a^0 \underset{6}{=} 1$

donc a^{-n} est l'inverse de a^n, $a^{-n} = \dfrac{1}{a^n}$

Donc 5 $\quad \dfrac{a^n}{a^m} = a^{n-m}$