1.1.5 03.09.19

b) $\sum_{j=1}^{2011}(-1)^j = (-1)^1 + (-1)^2 + --- + (-1)^{2011}$

$= \underbrace{-1 + 1}_{①\ ②} + .--- \underbrace{+ 1}_{2010} \underbrace{+ -1}_{2011} = -1$

c) $\sum_{k=-n}^{n}(k+1) = (-n+1) + (-n+1+1) + \ldots + 1 + --- + (n+1) = 2n+1$

$-n \qquad -n+1 \qquad 0$

$= (-n+1) + (-n+2) + \ldots + (n+1)$

$\boxed{n=3}$ $(\cancel{-3}+1) + (\cancel{-2}+1) + (\cancel{-1}+1) + (0+1) + (\cancel{1}+1) + (\cancel{2}+1) + (\cancel{3}+1) = 7$

$3 \qquad + \ 1 \ + \qquad 3$

h) $\sum_{k=1}^{6} \frac{1}{3^k}$

```
>>> from fractions import Fraction
>>> s = 0
>>> for k in range(1, 7):
        s+= Fraction(1,3**k)


>>> s
Fraction(364, 729)
```

1.1.6 Soit $x_1, \ldots, x_n$ une suite de nombres réels. Calculer $\sum_{i=2}^{n}(x_i - x_{i-1})$ $= x_n - x_1$

$$(\cancel{x_2} - x_1) + (\cancel{x_3} - \cancel{x_2}) + (x_4 - \cancel{x_3}) + \cdots + (\cancel{x_{n-1}} - x_{n-2}) + (x_n - \cancel{x_{n-1}})$$

$\overline{i=2}$

1.1.7 Traduire à l'aide du symbole Σ les sommes suivantes :

a) $1^2 + 2^2 + 3^2 + \cdots + 12^2 + 13^2 + 14^2$

b) $11^2 + 12^2 + 13^2 + \cdots + 102^2 + 103^2 + 104^2$

a) $\boxed{\sum_{K=1}^{14} K^2} = \sum_{j=2}^{15} (j-1)^2$

b) $\boxed{\sum_{i=11}^{104} i^2} = \sum_{i=1}^{94} (i+10)^2$

1.1.8 Ecrire les sommes suivantes à l'aide du symbole Σ.

a) $2+4+6+\cdots 248 =$

b) $1000+1010+1020+\cdots+1540 =$

c) $1^2+2^2+3^2+\cdots 15^2 =$

d) $1+2+4+8+16+\cdots+1024 =$

e) $2+3+5+9+17+\cdots+1025 =$

f) $4+12+36+108+324 =$

g) $9-12+15-18+\cdots+303 =$

h) $45-40+35-30+25-20+15 =$

f) $4\cdot 1 + 4\cdot 3 + 4\cdot 9 + 4\cdot 27 + 4\cdot 81 = \sum_{k=0}^{4} 4\cdot 3^k = 4\cdot\sum_{k=0}^{4} 3^k$

g) $3\cdot 3 - 3\cdot 4 + 3\cdot 5 - 3\cdot 6 + \cdots + 3\cdot 101 =$

$3(3-4+5-6+\cdots+101) = 3\cdot\sum_{k=3}^{101} \underbrace{(-1)^{k-1}}_{\text{alterne le signe}} k$

$$1+2+3+4+\dots+n-1+n = \sum_{K=1}^{n} K$$

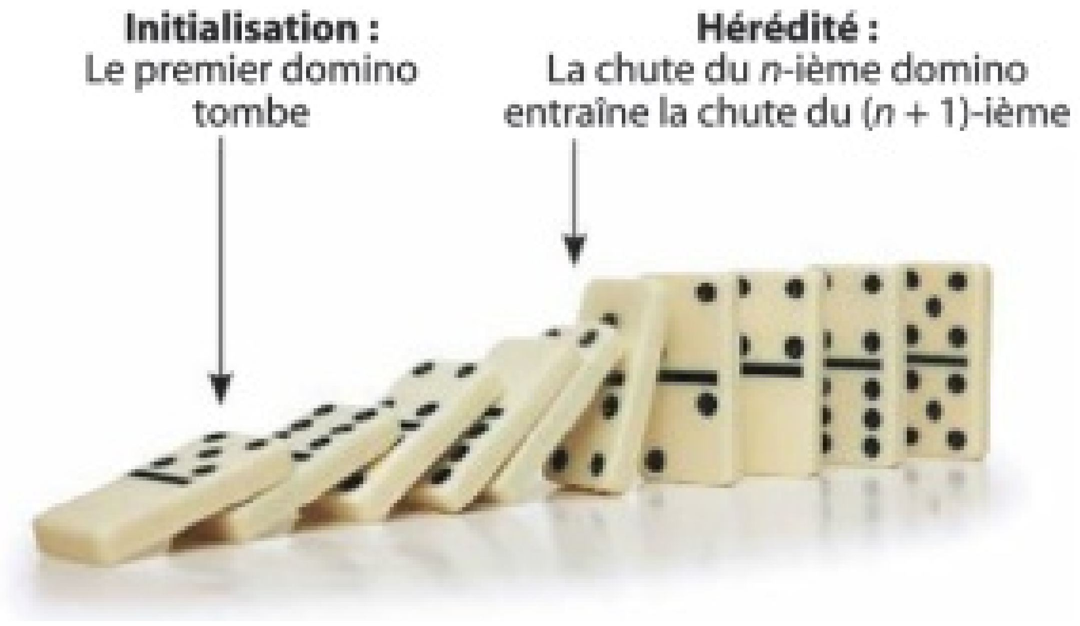
$$S_1 = \boxed{1} + \boxed{2} + 3 + 4 + \dots + \boxed{n-1} + \boxed{n}$$

$$S_1 = \boxed{n} + \boxed{n-1} + \dots + \boxed{2} + \boxed{1}$$

$$2S_1 = \underbrace{(n+1)+(n+1) \dots +(n+1)}_{n \text{ fois}} = n(n+1)$$

$$S_1 = \frac{n(n+1)}{2}$$

$$\boxed{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}}$$



Initialisation

1. $n = 1$ $\qquad 1 \;;\; \frac{1 \cdot 2}{2} = 1$ ✓

2. $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \Longrightarrow \sum_{k=1}^{n+1} \frac{(n+1)(n+2)}{2}$$

$$\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} k + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{n(n+1) + 2(n+1)}{2}$$

$$= \frac{n^2 + n + 2n + 2}{2} = \frac{n^2 + 3n + 2}{2} = \frac{(n+1)(n+2)}{2}$$