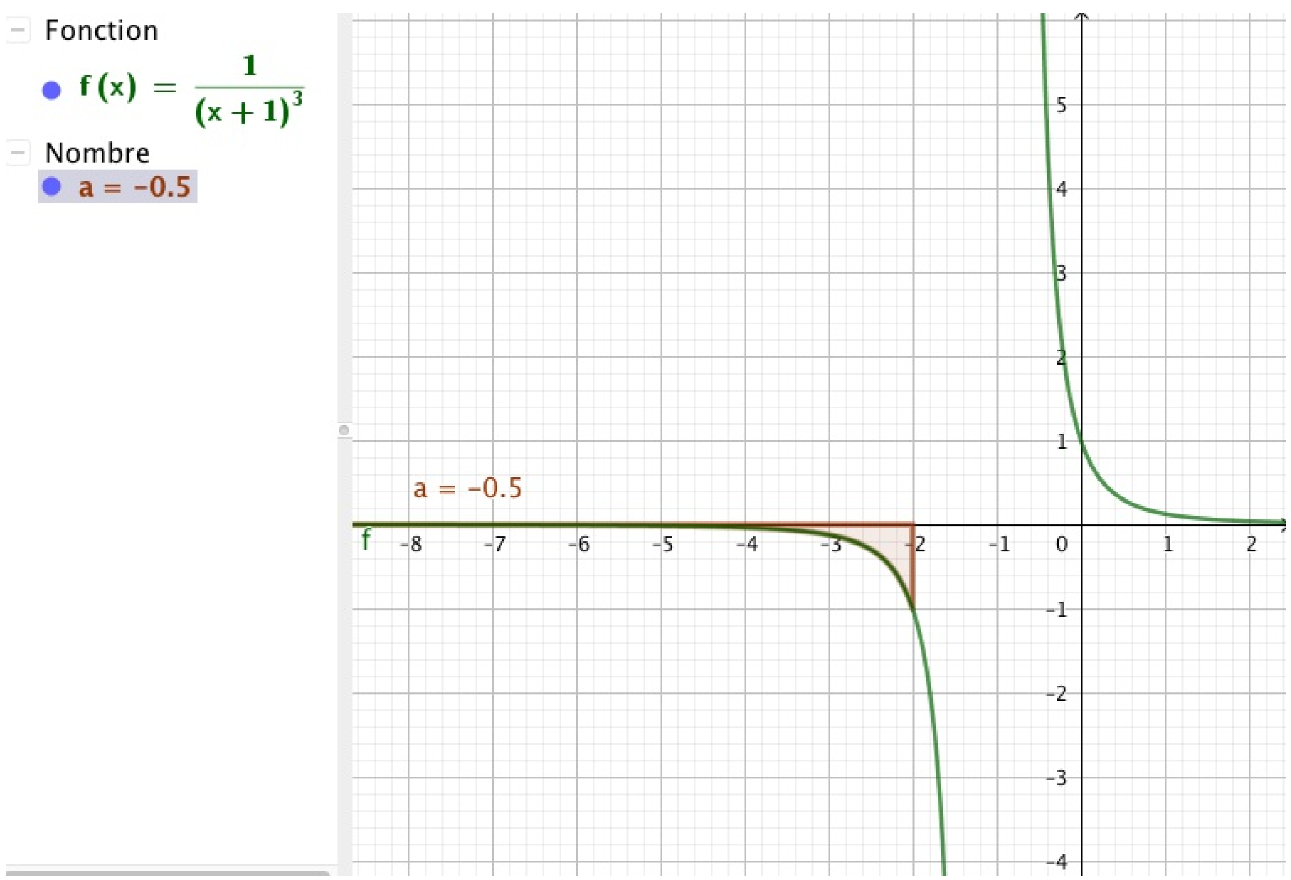
2.2.22

01.12.20

Intégrale généralisée

b) $\int_{-\infty}^{-2} \frac{1}{(x+1)^3}\,dx$

$$\lim_{K\to-\infty} \int_K^{-2} (x+1)^{-3}\,dx = \lim_{K\to-\infty}\left(\left[-\frac{1}{2}(x+1)^{-2}\right]_K^{-2}\right)$$



$$= \lim_{K\to-\infty}\left(-\frac{1}{2}\left((-1)^{-2} - (K+1)^{-2}\right)\right)$$

$$= \lim_{K\to-\infty} -\frac{1}{2}\left(1 - \underbrace{\frac{1}{(K+1)^2}}_{\to 0}\right)$$

$$= -\frac{1}{2}$$

d) $\int_0^{+\infty} \frac{x^2+1}{x^2}\,dx$

$ED(f) = \mathbb{R}^*$

$$\int_0^{+\infty} f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^{+\infty} f(x)\,dx$$

On calcule deux intégrales généralisées :

① $$\lim_{\substack{a \to 0 \\ >}} \left(\int_a^1 1 + x^{-2}\,dx \right) = \lim_{\substack{a \to 0 \\ >}} \left[x - x^{-1} \right]_a^1 =$$

$$\lim_{\substack{a \to 0 \\ >}} \left[x - \frac{1}{x} \right]_a^1 = \lim_{\substack{a \to 0 \\ >}} \left((1-1) - (a - \frac{1}{a}) \right)$$

$$\lim_{\substack{a \to 0 \\ >}} \frac{-a^2+1}{a} = +\infty$$

② $$\lim_{b \to +\infty} \left[x - x^{-1} \right]_1^b = \lim_{b \to +\infty} \left((b - \frac{1}{b}) - (1-1) \right)$$

$$= +\infty$$

Donc $$\int_0^{+\infty} f(x)\,dx = +\infty + \infty = +\infty$$

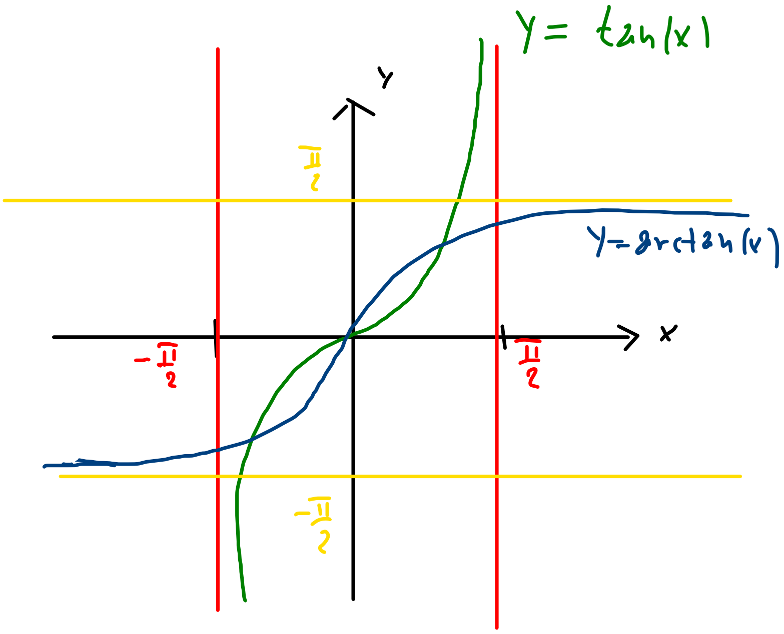
g) $\int_{-\infty}^{+\infty} \frac{3}{z^2+1}\,dz$ $\qquad ED(f) = \mathbb{R} \qquad f(x) > 0 \quad \forall x$

① $$\lim_{a \to -\infty} \int_a^0 \frac{3}{x^2+1}\,dx = 3 \cdot \lim_{a \to -\infty} \int_a^0 \frac{dx}{x^2+1} =$$

$$3 \cdot \lim_{a \to -\infty} \Big[\arctan(x)\Big]\Big|_a^0 = 3 \lim_{a \to -\infty} \big(0 - \arctan(a) \big)$$

$$= 3 \cdot \lim_{a \to -\infty} \big(-\arctan(a) \big)$$

$$= 3 \cdot -\left(-\frac{\pi}{2}\right) = \frac{3\pi}{2}$$



② On obtient également

$$3 \lim_{b \to +\infty} \int_0^b \frac{1}{1+x^2}\,dx = \frac{3\pi}{2}$$

Ainsi, l'intégrale généralisée est égale à $\frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

h) $\displaystyle\int_0^{\pi^2} \underbrace{\frac{\sin(\sqrt{x})}{\sqrt{x}}}_{f(x)}\,dx$ $\qquad ED(f) = \mathbb{R}_+^* =]0;+\infty[$

$$\lim_{a \to 0^+} \int_a^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \lim_{a \to 0^+} \int_{\sqrt{a}}^{\pi} \frac{\sin(t)}{\cancel{t}}\, 2\cancel{t}\,dt$$

changement de variables

$\phi(x) = \sqrt{x}$

$$\sqrt{x} = t \qquad t > 0$$
$$x = t^2$$
$$dx = 2t\,dt$$

x	$t = \phi(x)$
a	$\sqrt{a}$
π^2	π

$$= 2 \lim_{a \to 0^+} \int_{\sqrt{a}}^{\pi} \sin(t)\,dt = 2 \cdot \lim_{a \to 0^+} \left[-\cos(t)\right]_{\sqrt{a}}^{\pi}$$

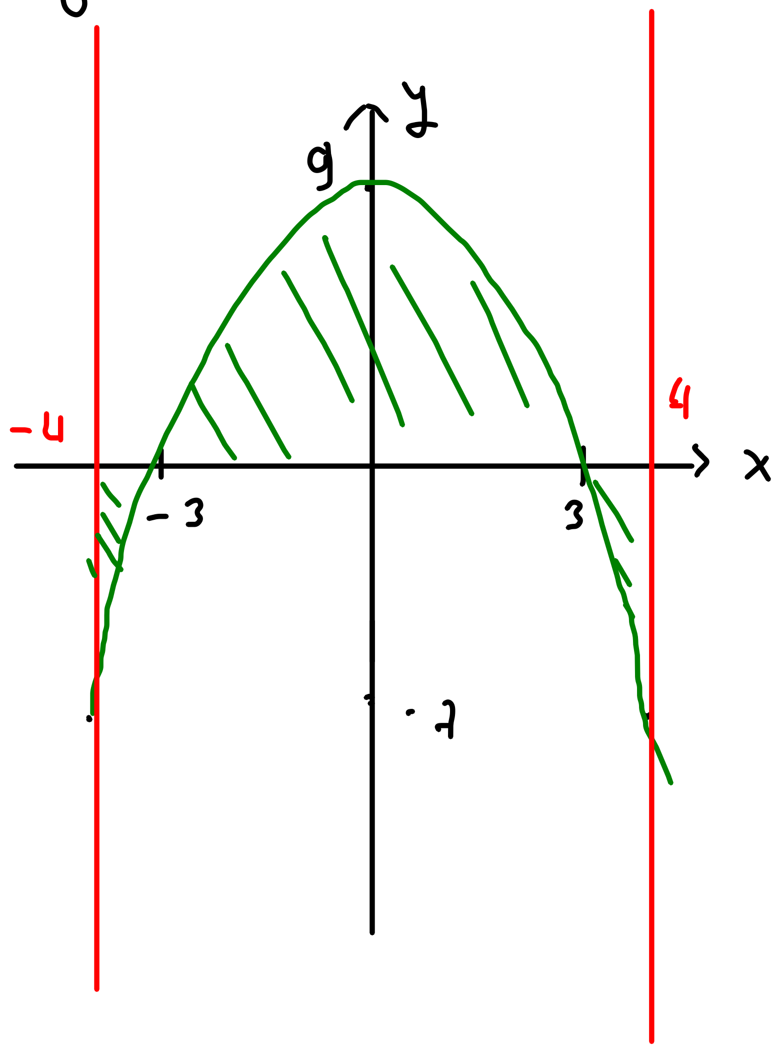
$$= -2 \lim_{a \to 0^+} \left(\cos(\pi) - \cos(\sqrt{a})\right) = -2\left(-1-1\right) = 4$$

2.2.24 Calculer l'aire du domaine limité par la courbe d'équation $y = f(x)$, l'axe Ox et les droites $x = a$, et $x = b$:

a) $f(x) = 9 - x^2, \quad a = -4, b = 4$;

$$f(x) = 9 - x^2 = (3-x)(3+x)$$

x		-3		3	
$f(x)$	$-$	0	$+$	0	$-$



$$\int_{-4}^{-3} f(x)\,dx = \left[9x - \frac{x^3}{3}\right]_{-4}^{-3} =$$ <0

$$\int_{-3}^{3} f(x)\,dx =$$ >0

$$\int_{3}^{4} f(x)\,dx =$$ <0