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b) $\int \sqrt{\sin(x)} \cos^3(x)\, dx$

d) $\int \frac{\cos(x)}{2 - \sin(x)}\, dx$

$\boxed{\text{On verra que } \int \frac{1}{x} dx = \ln|x| + c}$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} , \quad n \neq -1$$

d) changement de variable :

$$t = 2 - \sin(x)$$

$$dt = -\cos(x)\, dx$$

$$\int \frac{\cos(x)}{2-\sin(x)} dx = \int \frac{-dt}{t} = - \int \frac{1}{t} dt = - \ln|2-\sin(x)| + c$$

$$= - \ln(2-\sin(x)) + c$$

b) $\int \sqrt{\sin(x)} \cos^3(x)\, dx = \int \sqrt{\sin(x)} \cdot \cos(x) \cdot \cos^2(x)\, dx$

$$= \int \underbrace{\sqrt{\sin(x)}}_{(\sin(x))^{1/2}} \cdot \cos(x) \left(1 - \sin^2(x)\right) dx$$

$$= \int \left(\sin(x)\right)^{1/2} \cos(x)\, dx - \int \left(\sin(x)\right)^{5/2} \cos(x)\, dx$$

changement de variable $\quad t = \sin(x)$

$\quad dt = \cos(x)\, dx$

$$= \int t^{\frac{1}{2}}\, dt - \int t^{\frac{5}{2}}\, dt = \frac{1}{\frac{1}{2}+1} t^{\frac{1}{2}+1} - \frac{1}{\frac{5}{2}+1} t^{\frac{5}{2}+1} + c$$

$$= \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{7} t^{\frac{7}{2}} + c = \frac{2}{3} \sqrt{\sin^3(x)} - \frac{2}{7} \sqrt{\sin^7(x)} + C$$

c) $\int \sin(\underset{\beta}{5x}) \cos(\underset{\alpha}{3x})\, dx = \frac{1}{2} \int \left(\sin(8x) - \sin(-2x)\right) dx$

CRM (31): $\cos(\alpha) \sin(\beta) = \frac{1}{2} \left(\sin(\alpha + \beta) - \sin(\alpha - \beta) \right)$

$$= \frac{1}{2} \left[\int \sin(8x)\, dx + \int \sin(2x)\, dx \right]$$

$$= \frac{1}{2} \left(-\frac{1}{8} \cos(8x) - \frac{1}{2} \cos(2x) \right) + c$$

$$= \frac{-1}{16} \cos(8x) - \frac{1}{4} \cos(2x) + c$$

2.2.9 Trouver l'expression mathématique de la fonction f, sachant que :

a) $f'(x) = 3x^2 - 4$, $f(5) = 54$;

b) $f''(x) = (x+1)(x-2)$, $f(1) = 8$, $f'(0) = 37/6$;

c) $f''(x) = \frac{1}{\sqrt{x}}$, $f'(9) = 2$, $f(1) = 2f(4)$.

b) $f'(x) = \int (x+1)(x-2)\,dx = \int x^2 - x - 2\,dx$

$$= \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + c$$

• $f(0) = \frac{37}{6} \Rightarrow c = \frac{37}{6}$

$$f(x) = \int \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + \frac{37}{6}\,dx$$

$$= \frac{1}{12}x^4 - \frac{1}{6}x^3 - x^2 + \frac{37}{6}x + d$$

• $f(1) = 8$: $\frac{1}{12} - \frac{1}{6} - 1 + \frac{37}{6} + d = 8$

$$\frac{1 - 2 - 12 + 74 + 12d}{12} = 8$$

$$61 + 12d = 96$$

$$12d = 35 \Rightarrow d = \frac{35}{12}$$

$$\boxed{f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 - x^2 + \frac{37}{6}x + \frac{35}{12}}$$

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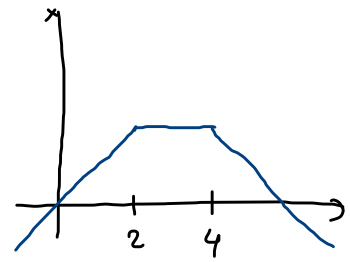
c) $f(x) = \begin{cases} x & \text{si } x < 2 \\ 2 & \text{si } 2 \leq x \leq 4, \\ 6 - x & \text{si } x > 4 \end{cases}$ avec $F(0) = 1$.



Cette fonction par morceaux est continue.
En effet,

a) $\lim\limits_{\substack{x \to 2 \\ <}} f(x) = 2$ et $\lim\limits_{\substack{x \to 2 \\ >}} f(x) = 2$

b) $\lim\limits_{\substack{x \to 4 \\ >}} f(x) = 2$ et $\lim\limits_{\substack{x \to 4 \\ <}} f(x) = 2$

Donc sa primitive est continue.

$$F(x) = \begin{cases} \frac{1}{2}x^2 + C_1 & x < 2 \\ 2x + C_2 & 2 \leq x \leq 4 \\ 6x - \frac{1}{2}x^2 + C_3 & x > 4 \end{cases}$$

Déterminons C_1, C_2 et C_3. On sait que $F(0) = 1$,
donc $C_1 = 1$

$$\lim_{\substack{x \to 2 \\ <}} F(x) = \lim_{\substack{x \to 2 \\ >}} F(x) \Rightarrow 3 = 4 + C_2 \Rightarrow C_2 = -1$$

$$\lim_{\substack{x \to 4 \\ <}} F(x) = \lim_{\substack{x \to 4 \\ >}} F(x) \Rightarrow 7 = 24 - 8 + C_3 \Rightarrow C_3 = -9$$

Finalement :

$$F(x) = \begin{cases} \frac{1}{2}x^2 + 1 & , x < 2 \\ 2x - 1 & , 2 \leq x \leq 4 \\ -\frac{1}{2}x^2 + 6x - 9 & , x > 4 \end{cases}$$

2.2.11 Déterminer la fonction f sachant qu'elle admet pour asymptote la droite

$$x - 2y + 8 = 0$$

et que

$$f''(x) = -\frac{8}{x^3}$$

On a une AO : $y = \frac{1}{2}x + 4$

$$f'(x) = \int -\frac{8}{x^3}\,dx = -8\int x^{-3}\,dx = -8 \cdot \frac{1}{-2}x^{-2} + c = \frac{4}{x^2} + c$$

$$f(x) = \int \left(4x^{-2} + c\right)dx = -4x^{-1} + cx + d = \frac{-4}{x} + cx + d$$

$$= \frac{cx^2 + dx - 4}{x}$$ donc $c = \frac{1}{2}$ et $d = 4$

Ainsi : $f(x) = \dfrac{\frac{1}{2}x^2 + 4x - 4}{x} = \dfrac{x^2 + 8x - 8}{2x}$