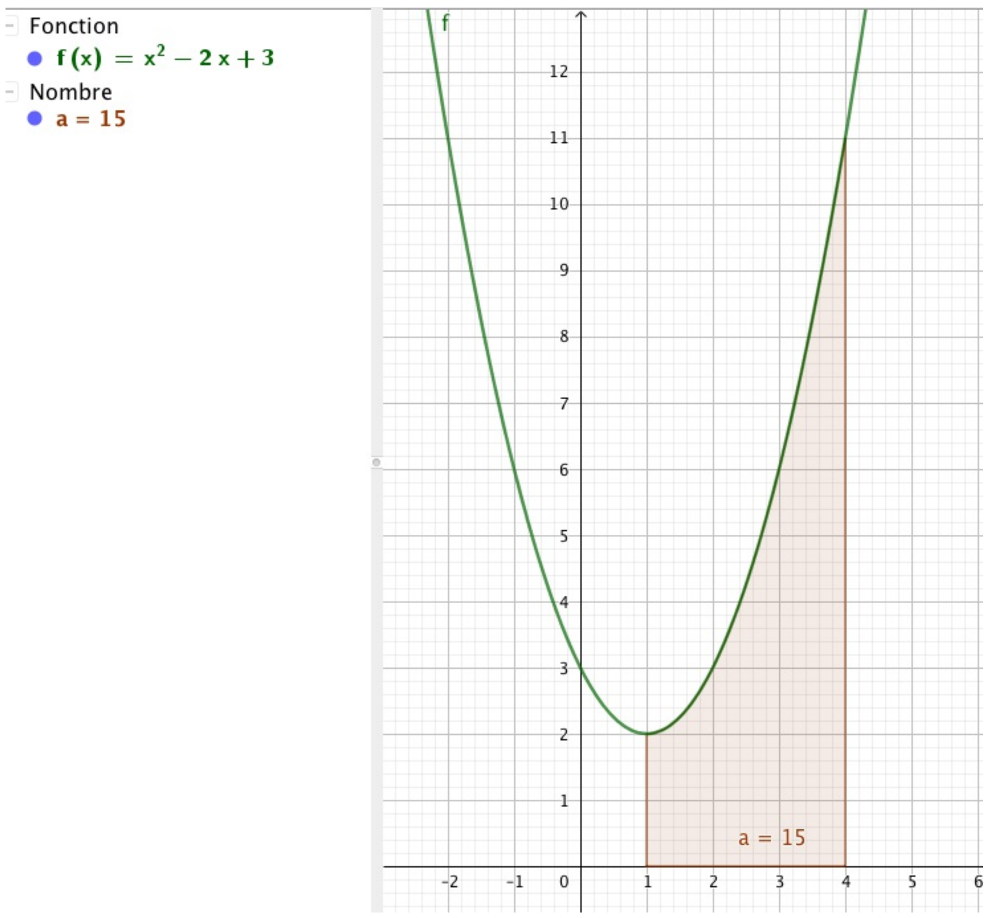
17.11.20

2.2.12 Calculer :

a) $\int_1^4 (x^2 - 2x + 3)\,dx = \left.\frac{1}{3}x^3 - x^2 + 3x\right|_1^4$

$= \left(\frac{1}{3}\cdot 64 - 16 + 12\right) - \left(\frac{1}{3} - 1 + 3\right)$

$= \frac{64}{3} - 4 - \frac{1}{3} - 2 = 15$



Propriétes de l'intégrale définie

Soit f une fonction continue et intégrable sur $[a,b]$.

Soit F une primitive de f.

1) $\displaystyle\int_a^b K\, f(x)\, dx = K\int_a^b f(x)\, dx \quad , \quad K \in \mathbb{R}$

2) $\displaystyle\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

3) $\displaystyle\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx \quad , \quad a \leq c \leq b$

Démonstrations:

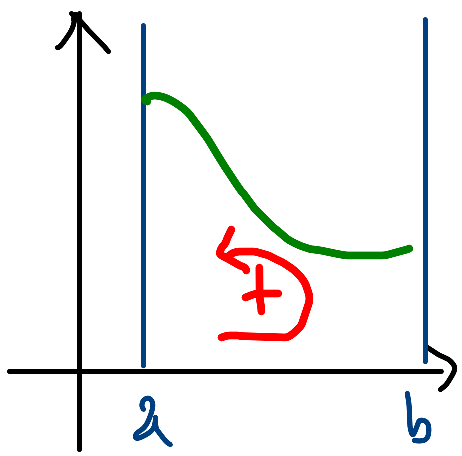
1) $\displaystyle\int K\, f(x)\, dx = K\int f(x)\, dx = K\, F(x) + d$

$$\int_a^b K\, f(x)\, dx = K\, F(x)\Big|_a^b = K F(b) - K F(a)$$

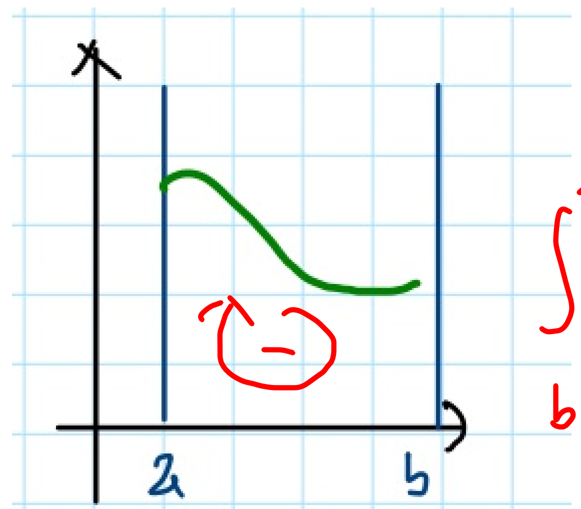
$$= K\left(F(b) - F(a)\right) = K\int_a^b f(x)\, dx$$

2) $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a) = -F(a) + F(b)$

$\displaystyle = -\big(F(a) - F(b)\big) = -F(x)\Big|_b^a = -\int_b^a f(x)\,dx$

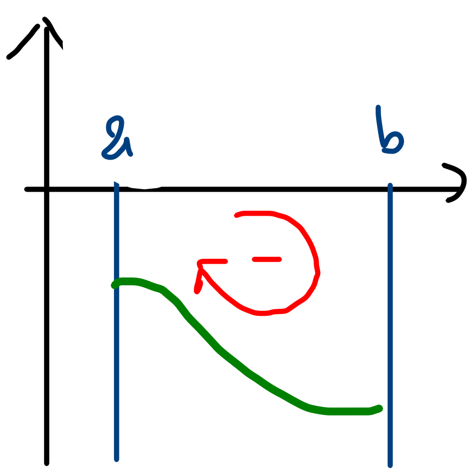


$\displaystyle\int_a^b f(x) > 0$

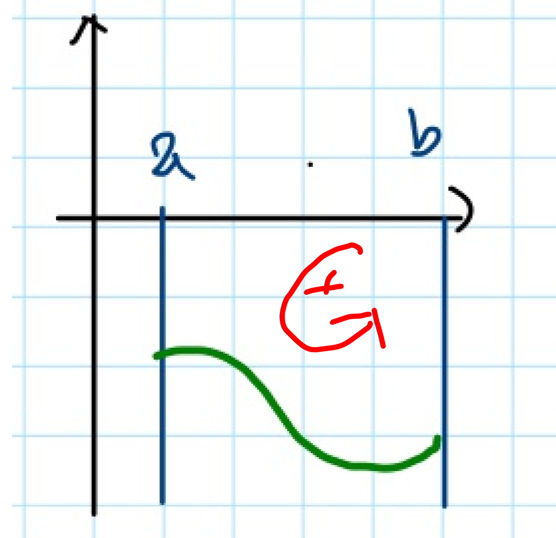


$\displaystyle\int_b^a f(x)\,dx < 0$

ii)



$\displaystyle\int_a^b f(x)\,dx < 0$



$\displaystyle\int_b^a f(x)\,dx > 0$

iii) $\displaystyle\int_1^2 -x^2\,dx = -\int_1^2 x^2\,dx = -\frac{x^3}{3}\Big|_1^2 = -\left(\frac{8}{3} - \frac{1}{3}\right) = -\frac{7}{3}$

3) $\int_a^b f(x)\,dx = F(b) - F(a) = F(b) - F(c) + F(c) - F(a)$

$$= \underline{F(x)\Big|_c^b} + \underline{F(x)\Big|_a^c}$$

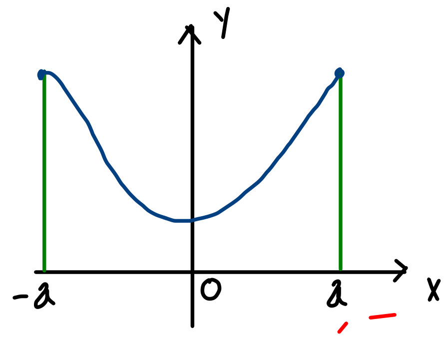
$$= \underline{\int_a^c f(x)\,dx} + \underline{\int_c^b f(x)\,dx}$$

2.2.14 Montrer que pour une fonction f continue sur $[-a;a]$, on a :

a) $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ lorsque f est paire ;

b) $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$ lorsque f est impaire.

a)



f est paire si $f(x) = f(-x)$

$a > 0$

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx = \int_{a}^{0} \underbrace{f(-t)}_{f(t)}(-dt) + \int_{0}^{a} f(x)\,dx$$

changement de variable

$t = -x$

$dt = -dx$

x	t
$-a$	a
0	0

$$= -\int_{a}^{0} f(t)\,dt + \int_{0}^{a} f(x)\,dx$$

$$= -\int_{a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= 2\int_{0}^{a} f(x)\,dx$$

b) $$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

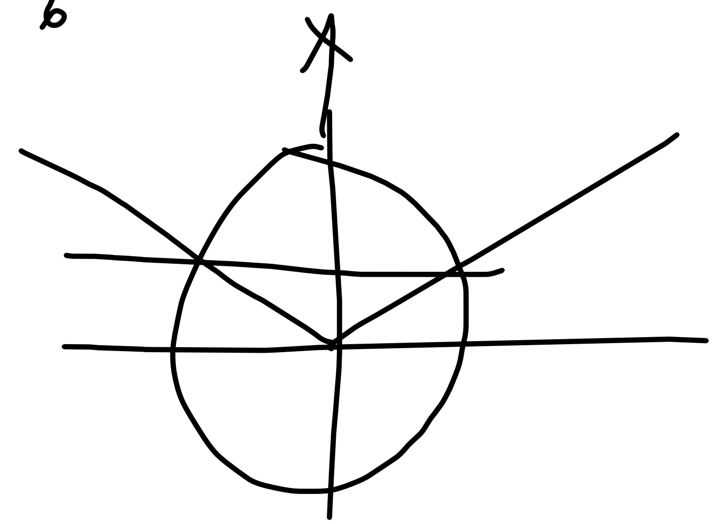
$$= \int_{a}^{0} \underbrace{f(-t)}_{-f(t)}(-dt) + \int_{0}^{a} f(x)\,dx$$

$$= \int_{a}^{0} f(t)\,dt + \int_{0}^{a} f(x)\,dx$$

$$= -\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= 0$$

$\sin\left(\frac{K}{2}\right) = \frac{1}{2}$ $\quad$ π $\Rightarrow$ $\frac{K}{2} = \frac{\pi}{6}$



$$\begin{cases} \frac{K}{2} = \frac{\pi}{6} + 2K\pi \\ \text{ou} \\ \frac{K}{2} = \frac{5\pi}{6} + 2K\pi \end{cases} \qquad K \in \mathbb{Z}$$