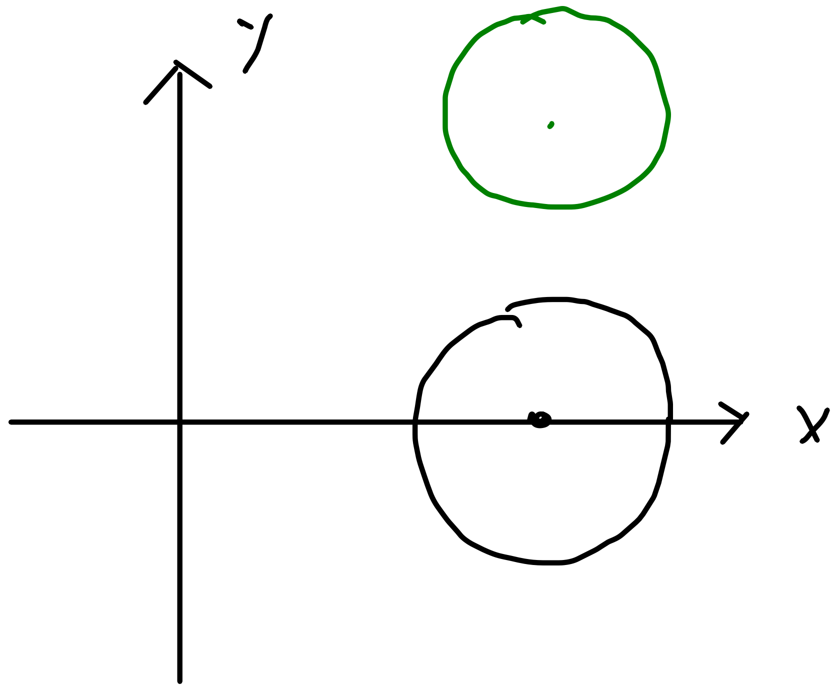
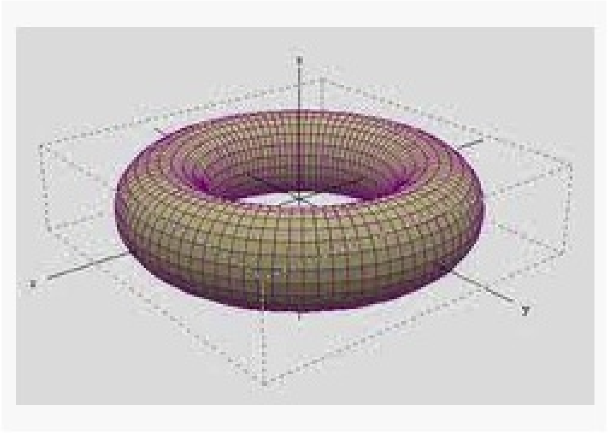
17.12.20

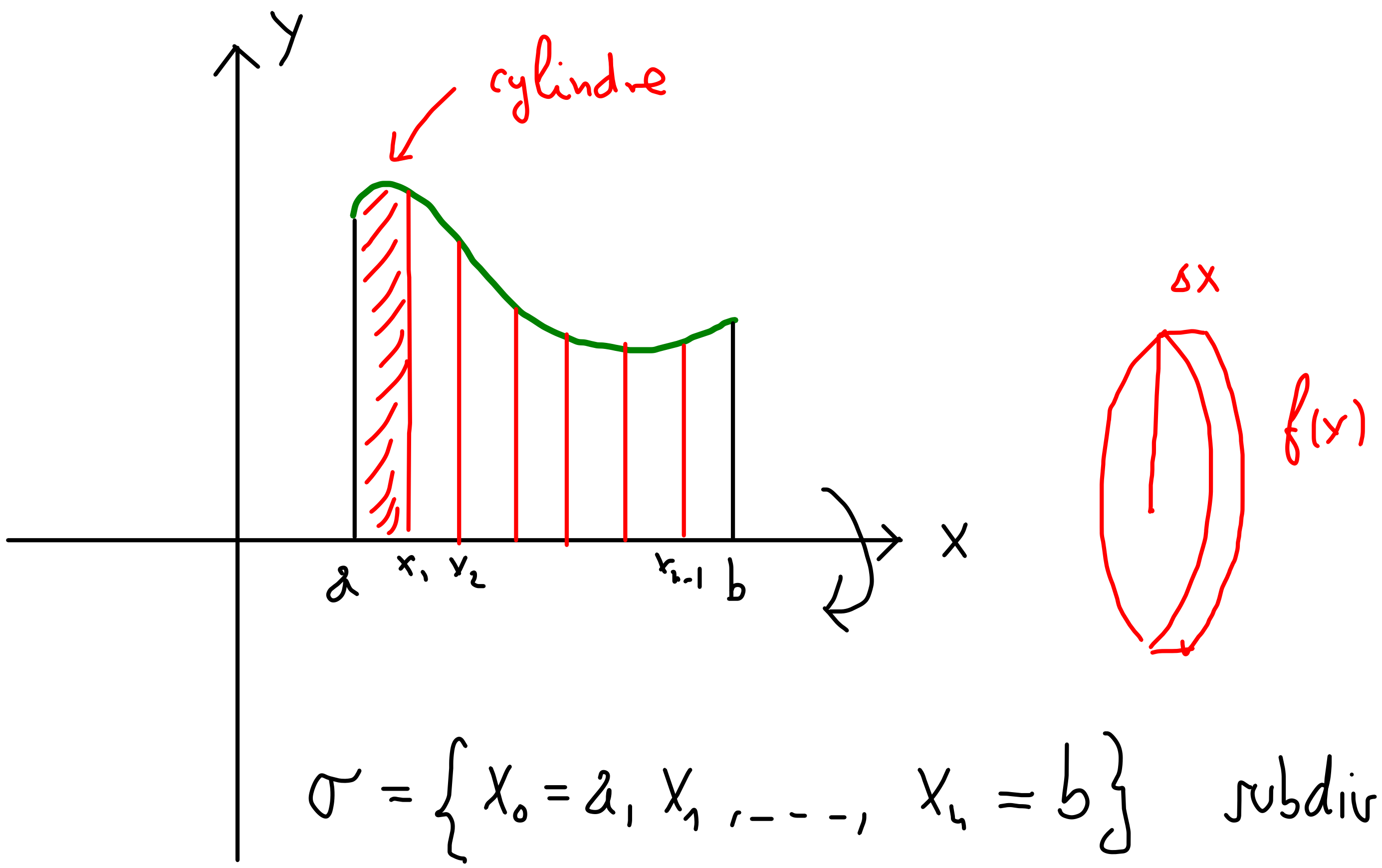
Solide de révolution



cercle centré sur O_x → sphère

cercle non centré sur O_x → tore





$\sigma = \{ x_0 = a, x_1, \ldots, x_n = b \}$ subdivision de $[a,b]$

$$\Delta x = \frac{b-a}{n}$$

Volume : $$V \cong \sum_{k=0}^{n} \pi \left(f(x_k) \right)^2 \Delta x$$

En passant à la limite,

$$\boxed{V = \pi \int_a^b \left(f(x) \right)^2 dx}$$

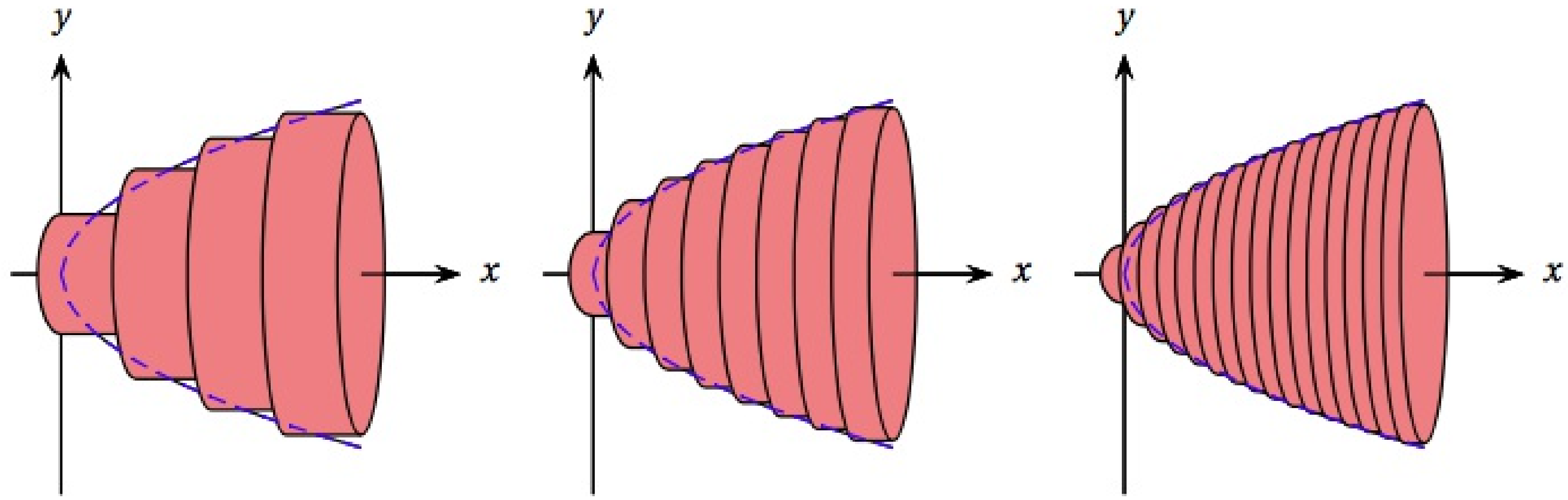
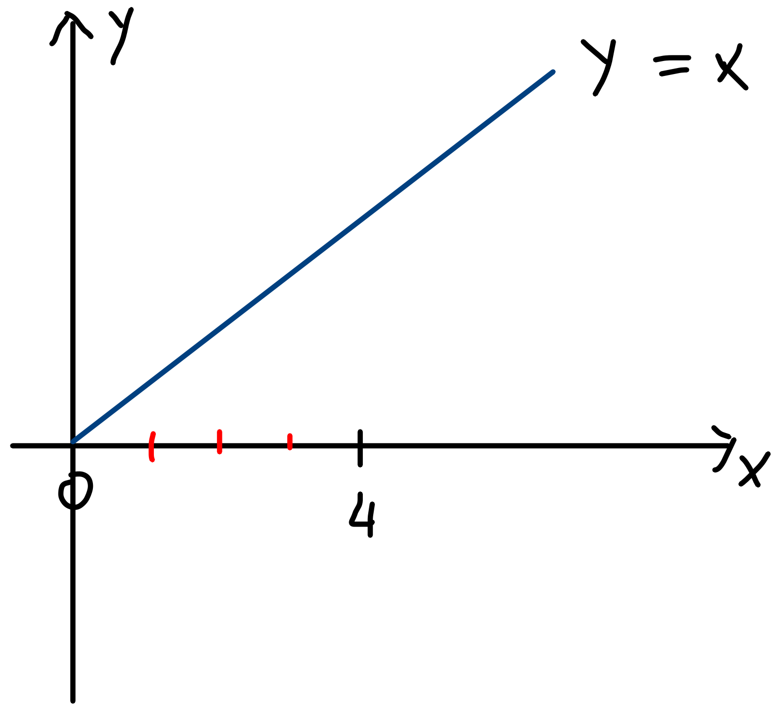
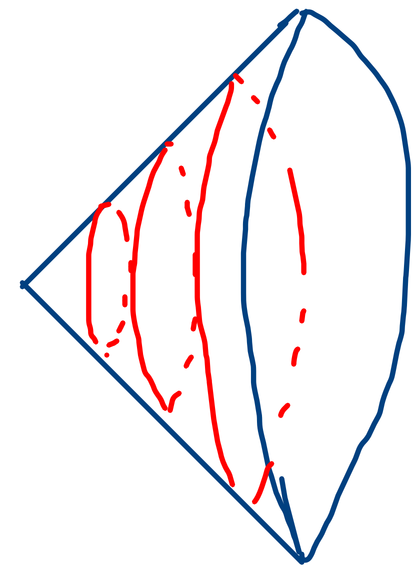


FIGURE 5.3 Approximation du solide S à l'aide d'une juxtaposition de disques.

Exemples

1)



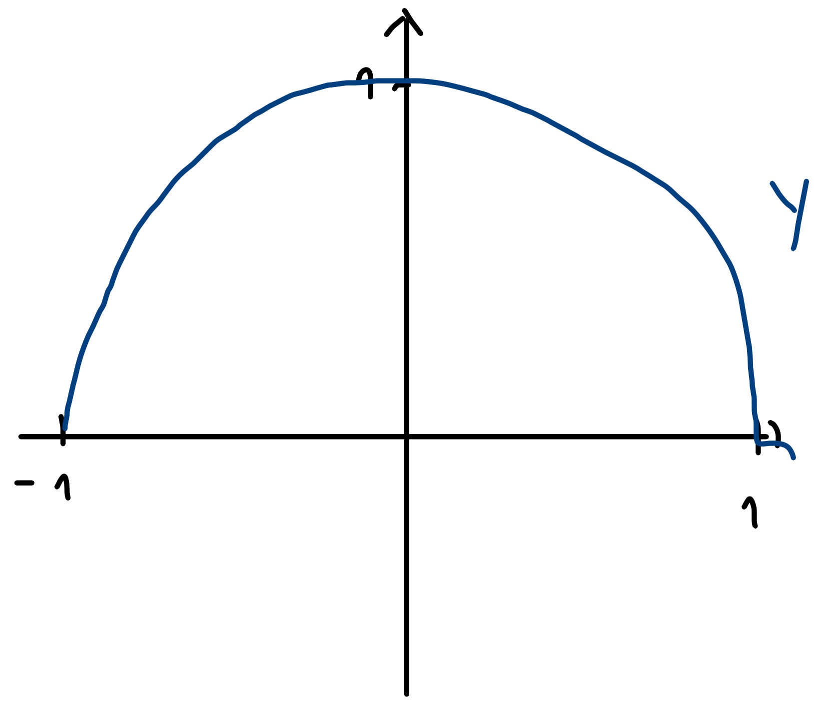


Cône de révolution

$$V = \frac{1}{3}\pi r^2 \cdot h = \frac{1}{3}\pi \cdot 4^2 \cdot 4 = \frac{64}{3}\pi$$

$$V = \pi \int_0^4 (x)^2 \, dx = \pi \left. \frac{x^3}{3} \right|_0^4 = \frac{64}{3}\pi$$

2)



$y = \sqrt{1-x^2}$

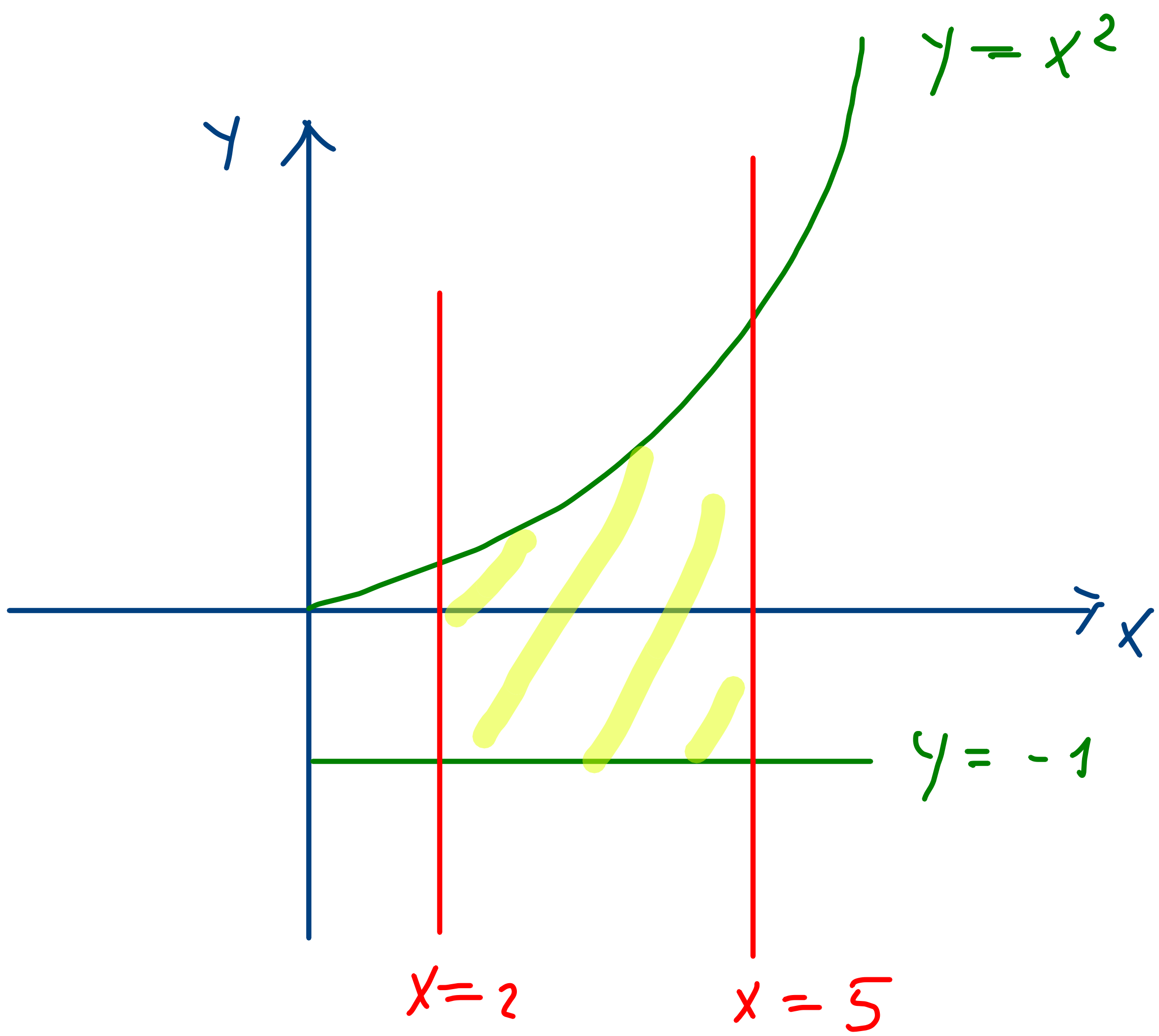
Sphère $\quad V = \frac{4}{3}\pi$

$$V = \pi \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx = \pi \int_{-1}^{1} 1-x^2 \, dx =$$

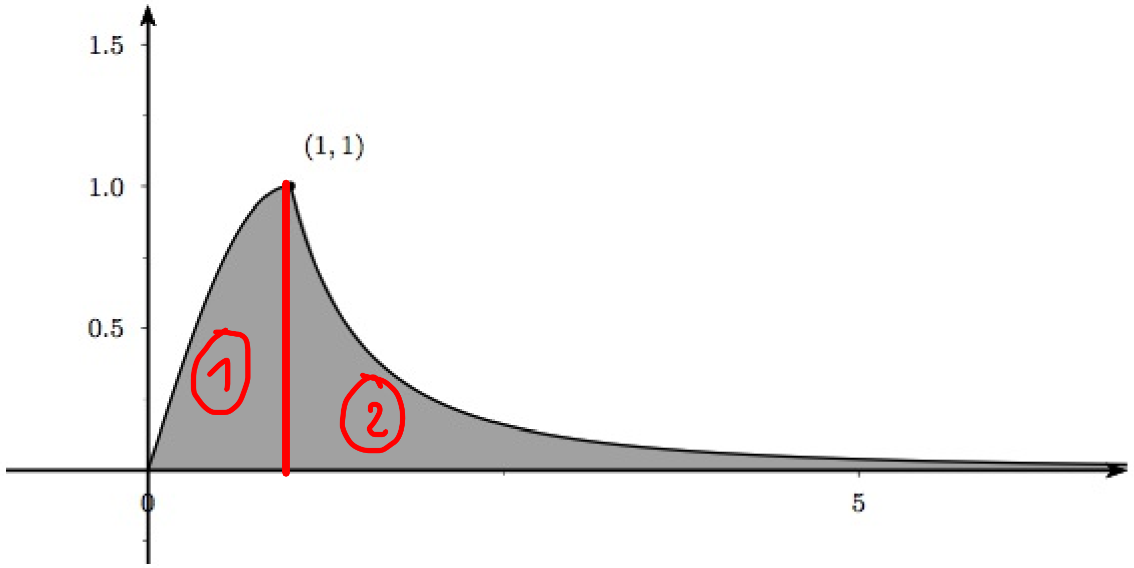
$$= \pi \left[x - \frac{x^3}{3}\right]_{-1}^{1} = \pi \left(\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right) \right) = \pi \cdot \frac{4}{3}$$

2.2.29 Calculer l'aire du domaine borné limité par les courbes données par les équations

$$y = x^2, \quad y = -1, \quad x = 2 \quad \text{et} \quad x = 5$$

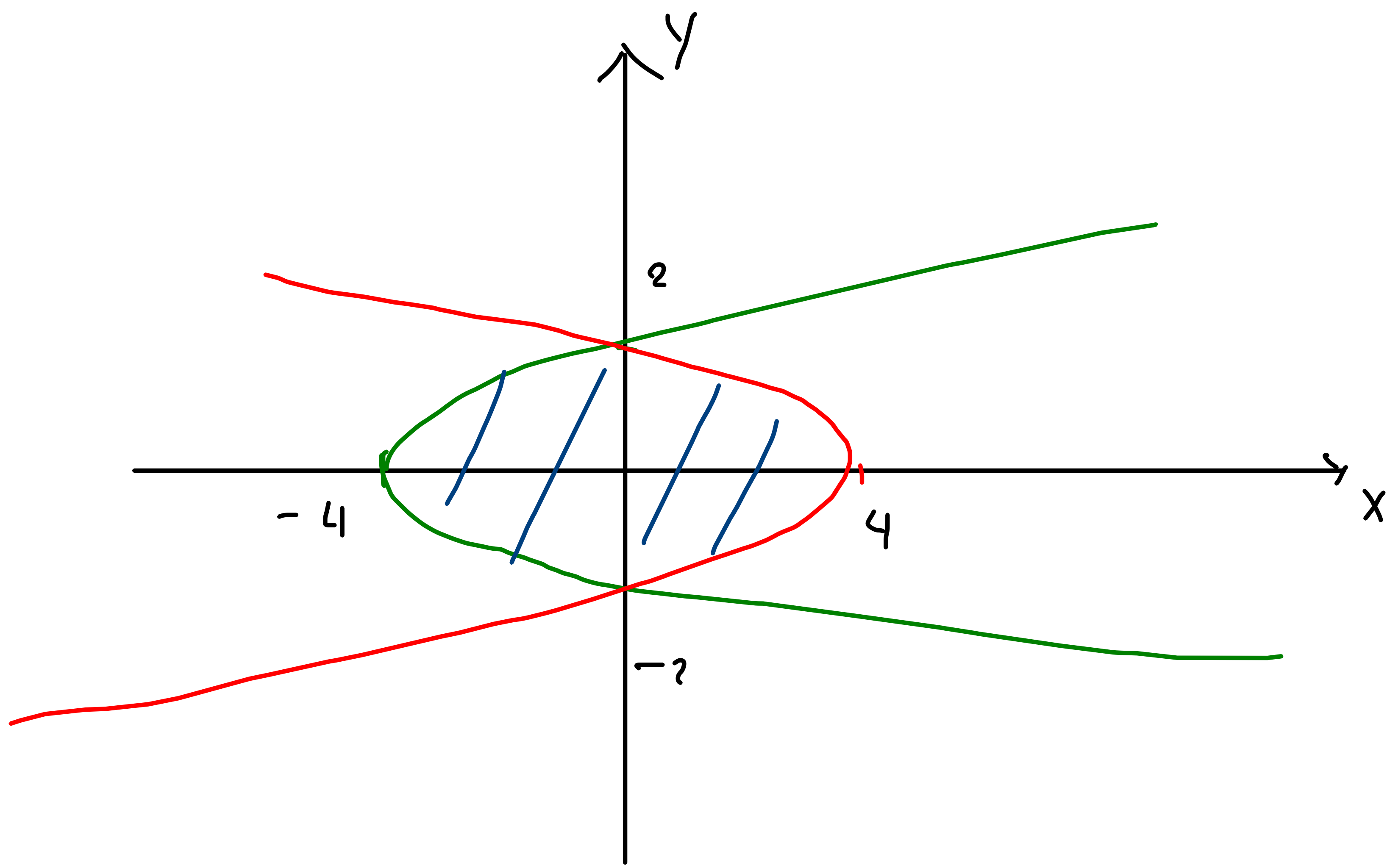


d) $y = \sin(ax)$, $y = \frac{1}{x^2}$



2.2.30 Calculer l'aire du domaine borné limité par les courbes données par les équations

$$y^2 = 4 - x \quad \text{et} \quad y^2 = 4 + x$$



$$y^2 = 4 - x \iff x = 4 - y^2 \qquad \mathcal{C}_1$$

$$y^2 = 4 + x \iff x = y^2 - 4 \qquad \mathcal{C}_2$$