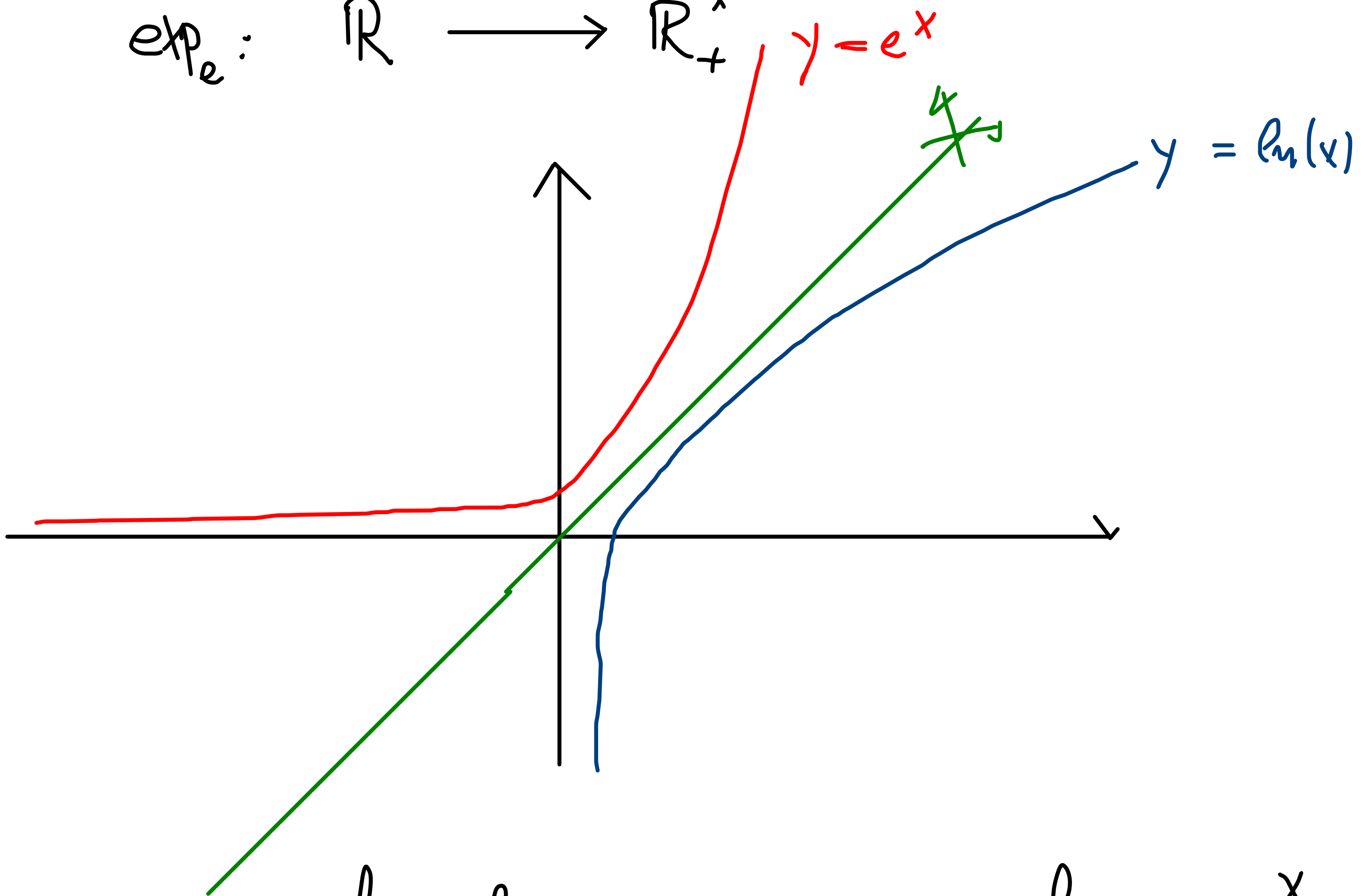
21.01.21

$\ln : \mathbb{R}_+^* \longrightarrow \mathbb{R}$

$\exp_e : \mathbb{R} \longrightarrow \mathbb{R}_+^*$



$\lim\limits_{x \to 0^+} \ln(x) = -\infty$ $\qquad$ $\lim\limits_{x \to -\infty} e^x = 0$

$\lim\limits_{x \to +\infty} \ln(x) = +\infty$ $\qquad$ $\lim\limits_{x \to +\infty} e^x = +\infty$

$\ln(1) = 0 \iff e^0 = 1$

2.3. 1

a) $f(x) = e^{5x}$ $\qquad ED(f) = \mathbb{R}$

b) $f(x) = e^{x^2}$ $\qquad ED(f) = \mathbb{R}$

c) $f(x) = e^{1/x}$ $\qquad ED(f) = \mathbb{R}^*$

d) $f(x) = e^{\sqrt{x^2+x}}$ $\qquad ED(f) =]-\infty ; -1] \cup [0 ; +\infty[$

Etudions le signe de $x^2+x = x(x+1)$

x		-1		0	
x^2+x	$+$	0	$-$	0	$+$

$$(e^u)' = u' \cdot e^u$$

c) $\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$; $f'(x) = -\frac{1}{x^2} e^{\frac{1}{x}}$

2.3.1

e) $f(x) = \exp\left(\sqrt{\frac{1+x^2}{1-x^2}}\right)$

Recherche de $ED(f)$:

x		-1		1	
$\frac{1+x^2}{1-x^2}$	$-$	||	$+$	||	$-$

$ED(f) =]-1;1[$

$$f'(x) = e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \left(\sqrt{\frac{1+x^2}{1-x^2}}\right)' = e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{4x}{(1-x^2)^2} \; \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}$$

1 $$\left(\frac{1+x^2}{1-x^2}\right)' = \frac{2x(1-x^2)-(1+x^2)\cdot(-2x)}{(1-x^2)^2} = \frac{2x\left(1-x^2+1+x^2\right)}{(1-x^2)^2}$$

$$= \frac{4x}{(1-x^2)^2}$$

2 $$\left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$$

$$\boxed{f'(x) = e^{\sqrt{\frac{1+x^2}{1-x^2}}} \cdot \frac{2x}{(1-x^2)^2} \sqrt{\frac{1-x^2}{1+x^2}}}$$

2.3.2 Calculer la dérivée d'ordre n de $f(x) = x\,e^x$.

$$f(x) = x e^x$$

$$f'(x) = e^x + xe^x = (1+x)e^x = e^x + f(x)$$

$$f''(x) = e^x + f'(x) = e^x + e^x + xe^x = (2+x)e^x$$

$$= 2e^x + f(x)$$

$$f'''(x) = 2e^x + f'(x) = 3e^x + f(x) = (3+x)e^x$$

$$f^{(n)}(x) = (n+x)e^x$$

$$e^0 = 1$$

$$e^1 = e$$

$$e^{\ln(x)} = x$$

$$\ln(e^x) = x$$

$$e^2 + e^2 = 2e^2$$

$$e^2 + e^3 = e^2(1+e)$$

2.3.4 Calculer les intégrales suivantes :

a) $\int_1^2 e^x\,dx$ $= e^x \Big|_1^2 = e^2 - e$

b) $\int_1^2 e^{3x-7}\,dx$ $= \frac{1}{3} e^{3x-7} \Big|_1^2 = \frac{1}{3}\left(e^{-1} - e^{-4}\right) = \frac{1}{3}\left(\frac{1}{e} - \frac{1}{e^4}\right)$

$= \frac{1}{3}\,\frac{e^3-1}{e^4} = \frac{e^3-1}{3e^4}$

c) $\int_0^2 x\,e^{x^2}\,dx$

$$\frac{1}{2}\int_0^2 2x e^{x^2}\,dx = \frac{1}{2} e^{x^2}\Big|_0^2 = \frac{1}{2}\left(e^4 - 1\right)$$

$$\lim_{x \to 0^{>}} x\, e^{\frac{1}{x}} \underset{0 \cdot +\infty}{\overset{\text{ind}}{=}} \lim_{t \to +\infty} \frac{1}{t} e^{t} = \lim_{t \to +\infty} \frac{e^{t}}{t}$$

$$x = \frac{1}{t} \iff t = \frac{1}{x}$$

2.3.4

d) $\int_1^2 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx = \int_1^{\sqrt{2}} \frac{1}{\cancel{t} e^{t}}\, 2\cancel{t}\, dt = 2\int_1^{\sqrt{2}} e^{-t}\, dt$

$t = \sqrt{x}$ $\qquad 1 \leq x \leq 2$

$t^2 = x$ $\qquad 1 \leq t\ \sqrt{2}$

$2t\, dt = dx$

x	t
1	1
2	$\sqrt{2}$

$= -2\, e^{-t} \Big|_1^{\sqrt{2}} = -2\, e^{-\sqrt{2}} + 2\, e^{-1}$

$= -2\left(e^{-\sqrt{2}} - e^{-1}\right)$

$$\lim_{x \to a} \frac{f(x)}{g(x)} \overset{\text{ind}}{\underset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} \overset{\text{ind}}{\underset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim_{x \to a} \frac{f'(x)}{g'(x)}$$