25.11.20

Formules

$$\boxed{\begin{aligned} \int \frac{1}{x}\,dx &= \ln|x| + c \\ \int \frac{u'}{u}\,du &= \ln|u| + c \end{aligned}}$$

$$\ln(x)+\ln(y) = \ln(xy)$$

$$\ln\left(\frac{1}{x}\right) = -\ln(x)$$

$$\ln(x)-\ln(y) = \ln\left(\frac{x}{y}\right)$$

$$k\ln(x) = \ln(x^k)$$

2.2.19 Calculer les intégrales définies suivantes.

a) $\int_1^2 \frac{x}{x+6}\,dx$

e) $\int_0^3 \sqrt{9-x^2}\,dx$

$$f(x) = \frac{x}{x+6} = 1 + \frac{-6}{x+6}$$

$$\begin{array}{r|l} x & x+6 \\ \cline{2-2} -\ x+6 & 1 \\ \cline{1-1} -6 & \end{array}$$

a) $$\int_1^2 \frac{x}{x+6}\,dx = \int_1^2 \left(1 - \frac{6}{x+6}\right)dx$$

$$= \int_1^2 dx - 6\int_1^2 \frac{1}{x+6}\,dx$$

$$= x\Big|_1^2 - 6\Big[\ln|x+6|\Big]_1^2 = (2-1) - 6\big(\ln(8) - \ln(7)\big)$$

$$= 1 - 6\ln\left(\frac{8}{7}\right)$$

f) $\displaystyle\int_2^3 \frac{5x-2}{x^2-x}\,dx = \int_2^3 \frac{5x-2}{x(x-1)}\,dx$

<u>Fraction rationnelle</u>

$$\frac{5x-2}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1} = \frac{a(x-1)+bx}{x(x-1)}$$

$$= \frac{ax+bx-a}{x(x-1)} = \frac{(a+b)x-a}{x(x-1)}$$

d'où le système $\begin{cases} a+b=5 \\ -a=-2 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=3 \end{cases}$

$$\int_2^3 \frac{2}{x}\,dx + \int_2^3 \frac{3}{x-1}\,dx = 2\Big[\ln|x|\Big]_2^3 + 3\Big[\ln|x-1|\Big]_2^3$$

$$= 2\big(\ln(3)-\ln(2)\big) + 3\big(\ln(2) - \underbrace{\ln(1)}_{0}\big)$$

$$= 2\ln(3) - 2\ln(2) + 3\ln(2)$$

$$= \ln(9) + \ln(8) - \ln(4)$$

$$= \ln(18)$$

g) $\int_{-1}^{0} \frac{dx}{x^2+2x+2} = \int_{-1}^{0} \frac{dx}{(x^2+2x+1)+1} = \int_{-1}^{0} \frac{dx}{(x+1)^2+1}$

$x^2+2x+2 = 0$ pas factorisable

$\Delta = 4-8 < 0$

changement de variables

$t = x+1$

$dt = dx$

x	t
-1	0
0	1

$= \int_{0}^{1} \frac{1}{t^2+1}\, dt \overset{\text{CRM}}{=} \arctan(t)\Big|_{0}^{1} = \frac{\pi}{4}$

2.2.22 Calculer, si possible, les intégrales généralisées ci-dessous :

a) $\displaystyle\int_1^{+\infty} \frac{2}{x^2}\,dx$

e) $\displaystyle\int_0^{2} \frac{2}{x^2}\,dx$

$$\int_1^{+\infty} \frac{2}{x^2}\,dx = \lim_{K \to +\infty} \underbrace{\left[\int_1^K \frac{2}{x^2}\,dx\right]} =$$

$$2\int_1^K x^{-2}\,dx = -2\,x^{-1}\Big|_1^K = -2\,\frac{1}{x}\Big|_1^K$$

$$= -2\left[\frac{1}{K} - 1\right]$$

$$= \frac{-2}{K} + 2$$

$$\lim_{K \to +\infty} 2 - \frac{2}{K} = 2$$

Fonction

- $f(x) = \frac{2}{x^2}$

Nombre

- $a = 2$
- $b = 2$