2.3.4 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 26.01.20

$$\int u'v = uv - \int uv'$$

f) $\displaystyle\int_1^{\ln(2)} x^2 e^x \, dx$

$u' = e^x dx \qquad u = e^x$

$v = x^2 \qquad v' = 2x\,dx$

$$\int x^2 e^x dx = x^2 e^x - 2 \underbrace{\int x e^x dx}$$

$u' = e^x dx \qquad u = e^x$

$v = x \qquad v' = dx$

$$= x^2 e^x - 2\left[x e^x - \int e^x dx \right] + C$$

$$= x^2 e^x - 2x e^x + 2e^x + C = (x^2 - 2x + 2) e^x + C$$

$$\int_1^{\ln(2)} x^2 e^x dx = \left(\ln^2(2) - 2\ln(2) + 2 \right) \cdot 2 - \left(1 - 2 + 2 \right) e =$$

Vérif

$$\left((x^2 - 2x + 2) e^x \right)' = (2x - 2) e^x + (x^2 - 2x + 2) e^x$$
$$= x^2 e^x$$

$$= 2\ln^2(2) - 4\ln(2) + 4 - e$$

2.3.5 changement de variable : $\frac{1}{x} = t \iff \frac{1}{t} = x$

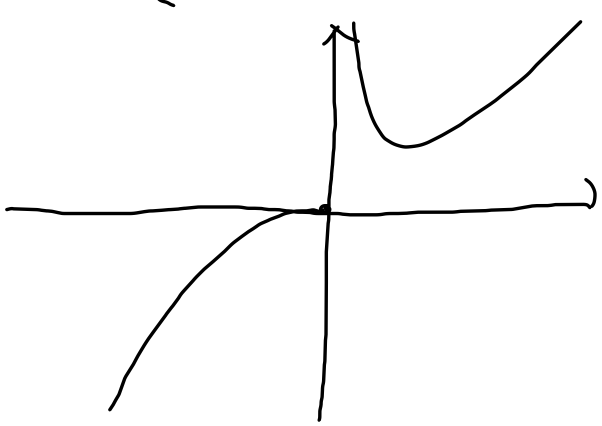
d) $\lim\limits_{\substack{x \to 0 \\ >}} x\, e^{1/x} \underset{"0_+ \cdot +\infty"}{\overset{ind}{=}} \lim\limits_{t \to +\infty} \frac{1}{t} \cdot e^{t} = \lim\limits_{t \to +\infty} \frac{e^{t}}{t} \underset{"\frac{+\infty}{+\infty}"}{\overset{BH}{=}} +\infty$

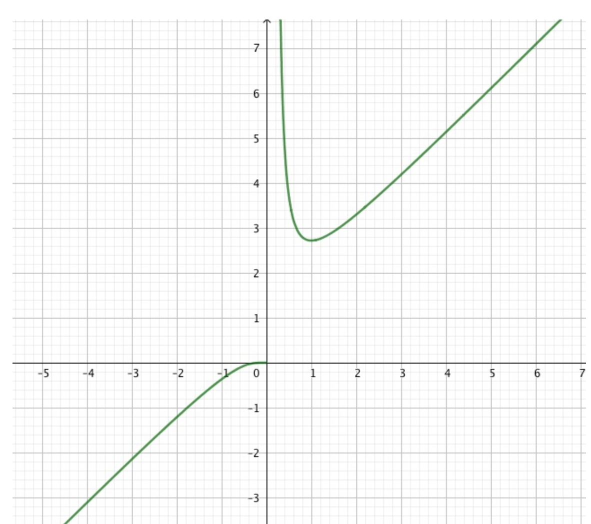
e) $\lim\limits_{x \to -\infty} \frac{e^{x} + e^{-x}}{2x} \underset{"\frac{0+\infty}{-\infty}"}{\overset{\substack{BH \\ ind}}{=}} \lim\limits_{x \to -\infty} \frac{e^{x} - e^{-x}}{2} \underset{"\frac{0-\infty}{2}"}{=} -\infty$

f) $\lim\limits_{x \to +\infty} \frac{2e^{x} - 1}{e^{x} + 2} \underset{"\frac{+\infty}{+\infty}"}{\overset{BH}{=}} \lim\limits_{x \to +\infty} \frac{2\cancel{e^{x}}}{\cancel{e^{x}}} = 2$

$$\boxed{\begin{array}{l} \lim\limits_{x \to +\infty} e^{x} = +\infty \\ \lim\limits_{x \to -\infty} e^{x} = 0 \end{array}}$$

d)bis $\lim\limits_{\substack{x \to 0 \\ <}} x\, e^{1/x} = \lim\limits_{t \to -\infty} \frac{e^{t}}{t} \overset{BH}{=} \lim\limits_{t \to -\infty} e^{t} = 0$





2, 3, 5

g) $\lim_{x\to-\infty}(x^2+x)\,e^x = \lim_{x\to-\infty}\frac{x^2+x}{e^{-x}} \overset{BH}{\underset{\text{"}\frac{+\infty}{+\infty}\text{"}}{=}}$

" $(+\infty-\infty)\cdot 0$

$= \lim_{x\to-\infty}\frac{2x+1}{-e^{-x}} \overset{BH}{\underset{\text{"}\frac{-\infty}{-\infty}\text{"}}{=}} \lim_{x\to-\infty}\frac{2}{e^{-x}} = 0$

2.3.9

[-1; 4]

$f(x) = \frac{1}{x}$

c) $\int_{-1}^{4} \frac{dx}{x}$

Intégrale généralisée

$ED(f) = \mathbb{R}^*$

<u>intégrale non définie</u>

d) $\int_{1}^{4} \frac{dx}{2x+3}$

c) $\int_{-1}^{4} \frac{dx}{x}$: $\lim_{t \to 0^<} \int_{-1}^{t} \frac{dx}{x} = \lim_{t \to 0^<} \left[\ln|x| \Big|_{-1}^{t} \right]$

$= \lim_{t \to 0^<} \ln|t| = \lim_{t \to 0^>} \ln(t) = -\infty$

$\lim_{t \to 0^>} \int_{t}^{4} \frac{dx}{x} = \lim_{t \to 0^>} \ln|x| \Big|_{t}^{4} = \lim_{t \to 0^>} \ln(4) - \ln(t)$

$= +\infty$

e) $\displaystyle\int_2^6 \frac{8x^3+19x^2+15x+4}{x^2+2x+1}\,dx$ f) $\displaystyle\int_{\pi/3}^{\pi/2} \frac{\sin(x)}{1-\cos(x)}\,dx$

e) Effectuons la division en colonne : $F(x) = 8x+3+\dfrac{x+1}{x^2+2x+1}$

$$
\begin{array}{rrrrr|l}
 & 8x^3 & +19x^2 & +15x & +4 & x^2+2x+1 \\
\cline{6-6}
- & 8x^3 & +16x^2 & +8x & & 8x+3 \\
\cline{2-5}
 & & 3x^2 & +7x & +4 & \\
 & - & 3x^2 & +6x & +3 & \\
\cline{3-5}
 & & & x & +1 &
\end{array}
$$

$$\int_2^6 f(x)\,dx = \int_2^6 (8x+3)\,dx + \int_2^6 \frac{\overset{1}{\cancel{x+1}}}{\underbrace{x^2+2x+1}_{(x+1)^{\cancel{2}}}}\,dx$$

$$= \Big[4x^2+3x\Big]_2^6 + \ln|x+1|\Big|_2^6$$

$$= 140 + \ln\left(\frac{7}{3}\right)$$

Remarque :

$$\int \frac{\overset{1}{\cancel{x+1}}}{(x+1)^{\cancel{2}}}\,dx = \ln|x+1| + c$$

$$\int \frac{x+1}{x^2+2x+2}\,dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+2}\,dx = \frac{1}{2}\ln\left|x^2+2x+2\right| + c$$

$u = x^2+2x+2$

$u' = 2(x+1)$

$$\int_a^b f(x)\,dx = \Big[F(x) + c\Big]_a^b = \big(F(b) + \cancel{c}\big) - \big(F(a) + \cancel{c}\big)$$

$$\int f(x)\,dx = F(x) + c$$

2.3.10 Calculer :

a) $\displaystyle\int \frac{4x-1}{x^2-2x-8}\,dx$

$$\frac{4x-1}{x^2-2x-8} = \frac{4x-1}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$$= \frac{A(x+2) + B(x-4)}{(x-4)(x+2)}$$

Donc $4x - 1 = A(x+2) + B(x-4)$

$4x - 1 = Ax + Bx + 2A - 4B$

$4x - 1 = (A+B)x + 2A - 4B$

Résolvons le système :

$$\begin{cases} A + B = 4 \\ 2A - 4B = -1 \end{cases} \Leftrightarrow \begin{cases} A = 5/2 \\ B = 3/2 \end{cases}$$

D'où l'intégrale :

$$= \int \frac{5/2}{x-4}\,dx + \int \frac{3/2}{x+2}\,dx = \frac{5}{2}\ln|x-4| + \frac{3}{2}\ln|x+2| + C$$

b) $\displaystyle\int \frac{1}{(x^2-1)^2}\,dx$

$$\frac{1}{(x-1)^2(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$$

éléments simples

$$1 = A(x-1)(x+1)^2 + B(x+1)^2 + C(x-1)^2(x+1) + D(x-1)^2$$

$$\begin{array}{ll} x=1 : & \\ x=-1 : & \\ x=0 & \\ x=2 & \end{array} \left\{ \begin{array}{lll} 1 = 4B & \Rightarrow & B = \frac{1}{4} \\ 1 = 4D & \Rightarrow & D = \frac{1}{4} \\ 1 = -A + \frac{1}{4} + C + \frac{1}{4} & & \\ 1 = 9A + \frac{9}{4} + 3C + \frac{1}{4} & & \end{array} \right.$$

$$\left\{ \begin{array}{l} B = 1/4 \\ D = 1/4 \\ -A + C = 1/2 \\ 9A + 3C = -3/2 \end{array} \right. \quad \begin{array}{|l} c \\ \\ \\ \cdot(-3) \\ \cdot\ 1 \end{array} \quad \left\{ \begin{array}{l} B = D = 1/4 \\ 12A = \frac{-6}{2} \\ C = 1/2 + A \end{array} \right. \iff \begin{array}{l} B = D = 1/4 \\ A = -1/4 \\ C = 1/4 \end{array}$$

$$\int \frac{1}{(x^2-1)^2}\,dx = -\frac{1}{4}\int \frac{dx}{x-1} + \frac{1}{4}\int\frac{dx}{(x-1)^2} + \frac{1}{4}\int\frac{dx}{x+1}$$

$$+ \frac{1}{4}\int \frac{dx}{(x+1)^2}$$

CRM

$= \ldots$