27.10.20

2.2.4 Calculer :

a) $\int \frac{dx}{x^2}$

d) $\int \sqrt{x}\, dx$

b) $\int \frac{2dx}{x^3}$

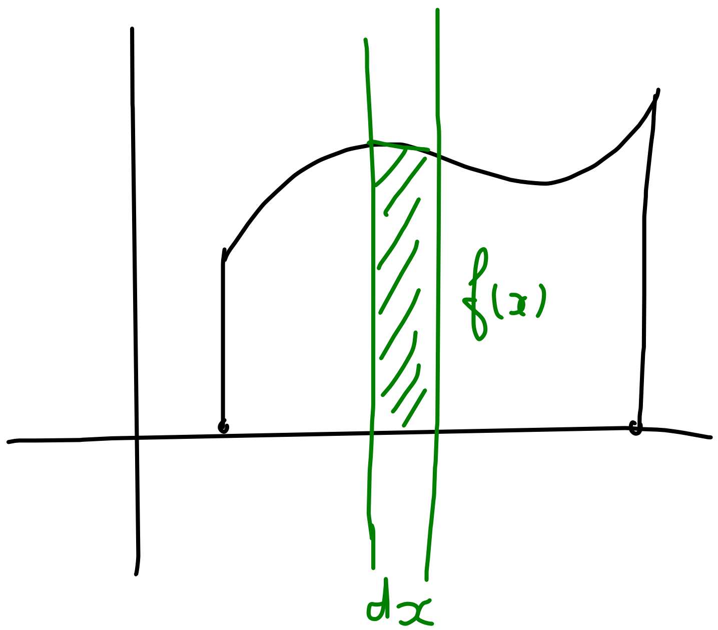
e) $\int \sqrt[3]{x}\, dx$

a) $\int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-1}}{-1} + c = \frac{-1}{x} + c$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad , n \neq -1$

e) $\int \sqrt[3]{x}\, dx = \int x^{\frac{1}{3}} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c = \frac{3}{4} x^{\frac{4}{3}} + c$

$= \frac{3}{4} \sqrt[3]{x^4} + c$



$$\sum \text{rectangle} = \int \underbrace{f(x)}_{\text{hauteur}}\ \overbrace{dx}^{\text{largeur}}$$

$$\int \underbrace{\sin(\cos(\tan(x)))}_{f(x)}\,dt = \sin(\cos(\tan(x)))\cdot t + c$$

$$\int \cos(t)\,\underline{x^2}\,dt = x^2\cdot\sin(t) + c$$

$$\int \cos(t)\,x^2\,dx = \cos(t)\cdot\frac{x^3}{3} + c$$

2.2.5 Calculer :

a) $\int \cos(3x)\,dx = \frac{1}{3}\sin(3x) + c$

$$\boxed{\int \cos(x)\,dx = \sin(x) + c}$$

$$\frac{d}{dx}\left(\sin(3x)\right) = \cos(3x)\cdot(3x)' = 3\cos(3x)$$

Changement de variable :

$$\int \cos(3x)\,dx = \int \cos(t)\,\frac{1}{3}dt = \frac{1}{3}\int \cos(t)\,dt$$

$$3x = t$$

$$3\,dx = dt \Rightarrow dx = \frac{1}{3}dt$$

$$= \frac{1}{3}\sin(t) + c$$

$$= \frac{1}{3}\sin(3x) + c$$

e) $\int (7x-2)^5\,dx = \frac{1}{42}(7x-2)^6 + c$

candidat : $K(7x-2)^6$

$$\left[K(7x-2)^6\right]' = K(7x-2)^5 \cdot 6 \cdot 7$$

$$= \underbrace{42K}_{1}(7x-2)^5 \quad \Rightarrow K = \frac{1}{42}$$

<u>changement de variable :</u>

$$7x-2 = t$$
$$7\,dx = dt \qquad \Rightarrow \quad dx = \frac{1}{7}dt$$

$$\int (7x-2)^5 dx = \int t^5 \frac{1}{7}dt = \frac{1}{7}\int t^5 dt = \frac{1}{7}\cdot\frac{t^6}{6} + c$$

$$= \frac{1}{42}t^6 + c = \frac{1}{42}(7x-2)^6 + c$$

f) $\int (3x^2+x)^3(6x+1)\,dx = \frac{1}{4}(3x^2+x)^4 + c$

dérivée interne

candidat : $K(3x^2+x)^4$

$$\left[K(3x^2+x)^4\right]' = K\cdot 4(3x^2+x)^3\cdot(6x+1)$$

$$= 4K(3x^2+x)^3(6x+1) \quad \Rightarrow K = \frac{1}{4}$$

Changement de variable

$$3x^2+x = t$$

$$(6x+1)\,dx = dt$$

$$\int (3x^2+x)^3(6x+1)\,dx = \int t^3\,dt = \frac{1}{4}t^4 + c$$

$$= \frac{1}{4}(3x^2+x)^4 + c$$

g) $\int (4x^2+3)^4 x\,dx = \frac{1}{40}(4x^2+3)^5 + c$

$$\Big[\underbrace{K(4x^2+3)^5}_{\text{candidat}}\Big]' = K \cdot (4x^2+3)^4 \cdot 5 \cdot 8x$$

$$= 40K\,(4x^2+3)^4\,x$$

i) $\int \frac{\tan^2(x)}{\cos^2(x)}\,dx = \int \frac{(\tan(x))^2}{\cos^2(x)}\,dx$

$(\tan(x))' = \frac{1}{\cos^2(x)}$

$\tan^2(x) = (\tan(x))^2$

$\tan(x^2)$

b) $\int \sin\left(2x - \frac{\pi}{3}\right) dx = -\frac{1}{2}\cos\left(2x - \frac{\pi}{3}\right) + c$

$$\Bigg(\underbrace{-\cos\left(2x - \frac{\pi}{3}\right)}_{\text{candidat}}\Bigg)' = \sin\left(2x - \frac{\pi}{3}\right) \cdot \left(2x - \frac{\pi}{3}\right)'$$

$$= 2\sin\left(2x - \frac{\pi}{3}\right)$$

c) $\int \cos^3(x)\,dx = \int \cos(x) \cdot \underline{\cos^2(x)}\,dx = \int \cos(x) \cdot \left(\underline{1 - \sin^2(x)}\right) dx$

$$= \int \cos(x) - \sin^2(x)\cos(x)\,dx$$

$$= \sin(x) - \frac{1}{3}\sin^3(x) + c$$

d) $\int \sin^4(x)\,dx = \int \left(1 - \cos^2(x)\right)^2 dx$

$$= \int \left(\sin^2(x)\right)^2 dx = \int \left(\frac{1 - \cos(2x)}{2}\right)^2 dx$$