0.1 (59)

1) $y = \frac{x^4}{16}$, $y' = \frac{x^3}{4}$

$y^{\frac{1}{2}} = \frac{x^2}{4}$

{équation différentielle du premier ordre}

Ainsi : $x \cdot y^{\frac{1}{2}} = x \cdot \frac{x^2}{4} = \frac{x^3}{4} = y'$

{ED du 2ème ordre}

2) $y = xe^x$

$y' = e^x + xe^x = (1+x)e^x$

$y'' = (2+x)e^x$

Ainsi $(2+x)e^x - 2(1+x)e^x + xe^x = [\underbrace{2+x-2-2x+x}]e^x$

$= 0$

0.2 (59)

$x^2+y^2=4$, pour $x \in]-2;2[$ est une solution implicite de l'ED $y' = \frac{-x}{y}$

En effet, dérivons la relation :

$$2x + 2yy' = 0$$

$$\Leftrightarrow \quad x + yy' = 0$$

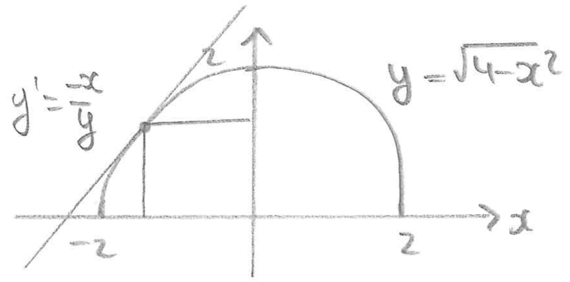
$$yy' = -x$$

$$y' = \frac{-x}{y}$$

(γ) : $x^2+y^2=4$ est un cercle de centre $(0,0)$ et de rayon 2.

ainsi, pour $y>0$ et $x \in]-2,2[$, on peut écrire

$$y=\sqrt{4-x^2}$$



$$y'=\frac{-2x}{2\sqrt{4-x^2}}=\frac{-x}{\sqrt{4-x^2}}=\frac{-x}{y}$$

0,3 (59)

ED de 2ème ordre : $y'' + 16y = 0$

$y_1 = C_1 \cos(4x)$ $y_1' = -4C_1 \sin(4x)$ $y_1'' = -16C_1 \cos(4x)$

$y_2 = C_2 \sin(4x)$ $y_2' = 4C_2 \cos(4x)$ $y_2'' = -16C_2 \sin(4x)$

$y = y_1 + y_2$

Avec y_1 : $-16C_1 \cos(4x) + 16C_1 \cos(4x) = 0$

Avec y_2 : $-16C_2 \sin(4x) + 16C_2 \sin(4x) = 0$

Comme $y_1'' + 16y_1 = 0$ et $y_2'' + 16y_2 = 0$, alors

$$\underbrace{y_1'' + y_2''}_{y''} + 16\underbrace{(y_1 + y_2)}_{y} = 0$$

Q.4 (59)

1) ED du 1er ordre : $y' - 3y = -3$

$y = \lambda e^{3x} + 1$ est une solution de l'équation.

$y' = 3\lambda e^{3x}$. Donc

$$y' - 3y = 3\lambda e^{3x} - 3(\lambda e^{3x} + 1)$$
$$= 3\lambda e^{3x} - 3\lambda e^{3x} - 3 = -3$$

2) ED du 1er ordre : $y'(x^2 - 1) = xy$

$c \neq 0 \quad x^2 + cy^2 = 1.$

En dérivant : $2x + 2cyy' = 0$

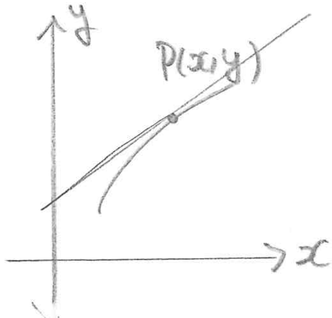
$$y' = \frac{-x}{cy}$$

Comme $x^2 = 1 - cy^2$, alors

$$y'(x^2-1) = \frac{-x}{cy}\left(1 - cy^2 - 1\right) = \frac{-x}{cy} \cdot (-cy^2) = xy$$

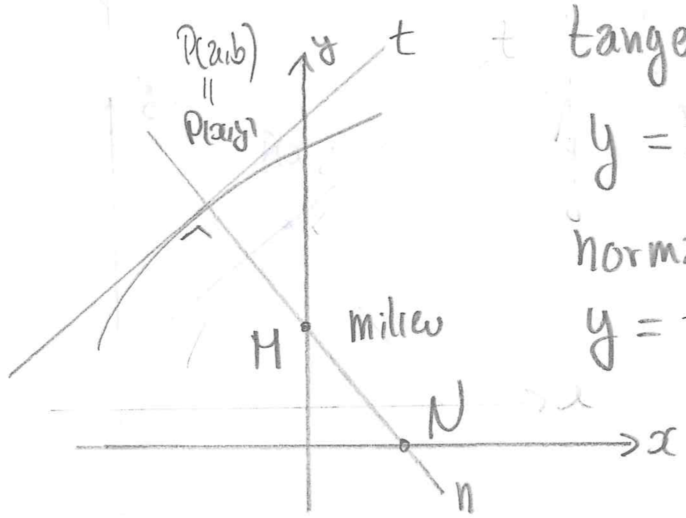
0,5 (59)

1)



$$y' = 2(x+y)$$

2)



tangente :

$$y = f'(a)(x-a) + f(a)$$

normale :

$$y = -\frac{1}{f'(a)} x + h$$

$$f(a) = -\frac{1}{f'(a)} a + h \Rightarrow h = f(a) + \frac{a}{f'(a)}$$

$$y = \frac{-x}{f'(a)} + \frac{a}{f'(a)} + f(a) \qquad x = a + f'(a) f(a)$$

$$N(a + f'(a) f(a) \,;\, 0)$$

$$P(a, f(a))$$

Le milieu de NP est sur Oy. Donc $y_M = 0$

$$\Rightarrow 2a + f'(a) f(a) = 0$$

$$\boxed{2x + y'y = 0}$$

0.6 (59)

$y' = ky$ avec $k \neq 0$

$\frac{y'}{y} = k$. On intègre

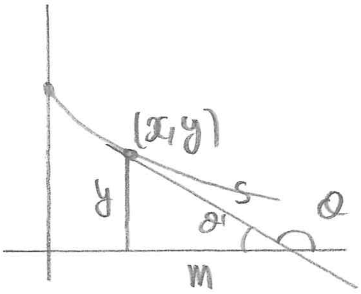
$\ln|y| = kx + c$

$y = e^{kx+c}$

$\underline{\underline{y = d\,e^{kx}}}$

0.7 (59)

$\tan(\pi-\theta) = \tan(\theta') = -\tan(\theta)$



$y' = \tan(\theta) = -\tan(\theta')$

$$\tan(\theta') = \frac{y}{m}$$

$$\parallel$$

$$-y' = \frac{y}{\sqrt{s^2-y^2}}$$

0.8 (60)

1) $y = \lambda e^{-x}$

$y' = -\lambda e^{-x}$ $\Rightarrow$ $y = -y'$ ou $\underline{\underline{y + y' = 0}}$

2) $y = \lambda \ln(x) + 1$

$y' = \frac{\lambda}{x}$

$y = \underbrace{\frac{\lambda}{x}}_{y'} \cdot x \ln(x) + 1 = y' x \ln(x) + 1$

$\underline{\underline{y = x y' \ln(x) + 1}}$

3) $y = \lambda_1 \sin(\omega t) + \lambda_2 \cos(\omega t)$

$y' = \lambda_1 \omega \cos(\omega t) - \lambda_2 \omega \sin(\omega t)$

$y'' = -\lambda_1 \omega^2 \sin(\omega t) - \lambda_2 \omega^2 \cos(\omega t)$

$= -\omega^2 (\lambda_1 \sin(\omega t) + \lambda_2 \cos(\omega t))$

$y'' = -\omega^2 y$ ou $\underline{\underline{y'' + \omega^2 y = 0}}$

4) $y = \lambda_1 e^{4t} + \lambda_2 e^{-4t}$

$y' = 4\lambda_1 e^{4t} - 4\lambda_2 e^{-4t}$

$y'' = 16\lambda_1 e^{4t} + 16\lambda_2 e^{-4t}$

$= 16(\lambda_1 e^{4t} + \lambda_2 e^{-4t})$

$y'' = 16y$ ou $\underline{\underline{y'' - 16y = 0}}$

5) $y = ax^2 + bx + c$

$y' = 2ax + b$

$y'' = 2a$

$\underline{\underline{y''' = 0}}$

0,9 (60)

1) $y = y'x$ ou $y - y'x = 0$

ou $\underline{\underline{xy' - y = 0}}$

2) (γ) : $(x-a)^2 + y^2 = a^2$

Dérivons : $2(x-a) + 2yy' = 0$

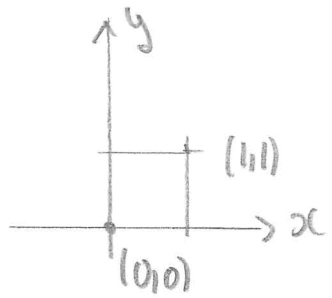
ou $x - a = -yy'$

et $\boxed{a = x + yy'}$

dans (γ) : $(yy')^2 + y^2 = (x + yy')^2$

$= x^2 + 2xyy' + (yy')^2$

et ainsi $2xyy' = y^2 - x^2$

3) 

$y = ax^2 + bx + c \Rightarrow c = 0$

et $1 = a + b \Rightarrow b = a - 1$

$y = ax^2 + (1-a)x$

Dérivons : $y' = 2ax + (1-a)$

$y' - 1 = a(2x-1) \Rightarrow a = \dfrac{y'-1}{2x-1}$

$$y = \frac{y'-1}{2x-1}\, x^2 + \left(1 - \frac{y'-1}{2x-1}\right) x$$

ou $$y(2x-1) = (y'-1)x^2 + (2x \underline{-1} - y' \underline{+1})\, x$$

$$y(2x-1) = (y'-1)x^2 + 2x^2 - y'$$

$$y(2x-1) = (x^2-1)y' - x^2 + 2x^2$$

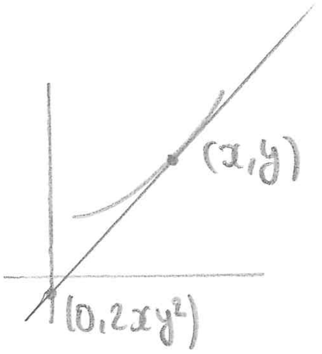
donc $$(x^2-1)y' - (2x-1)y + x^2 = 0$$

0.10 (60)

1) $P' = K(200'000 - P)$ où $P' = \frac{dP}{dt}$

2) $\frac{dp}{dT} = K\frac{P}{T^2}$

3)



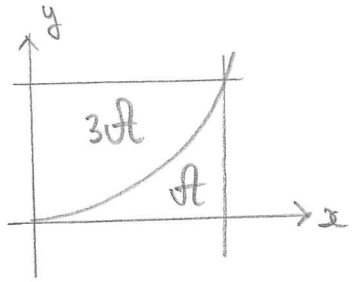
tangente: $y = f'(a)(x-a) + f(a)$

$y = f'(a)x + f(a) - f'(a)a$

$$2xy^2 = y - y'x$$

$$y'x = -2xy^2 + y$$

4)



$$xy = 4\int y\,dx$$

$$y + xy' = 4y$$

$$3y = xy'$$

$$\Rightarrow \quad y' = \frac{3y}{x}$$

0.11 (61)

$$\frac{P(t)}{dt} = K P(t) + m$$

0.12 (61)

J'aurais dit $\frac{A(t)}{M} = K(M - A(t))$

Mais $A'(t) = K(M - A(t))$

0.13 (61)

$$mg - Kx = ma$$

$$mg - Kx = m x''$$