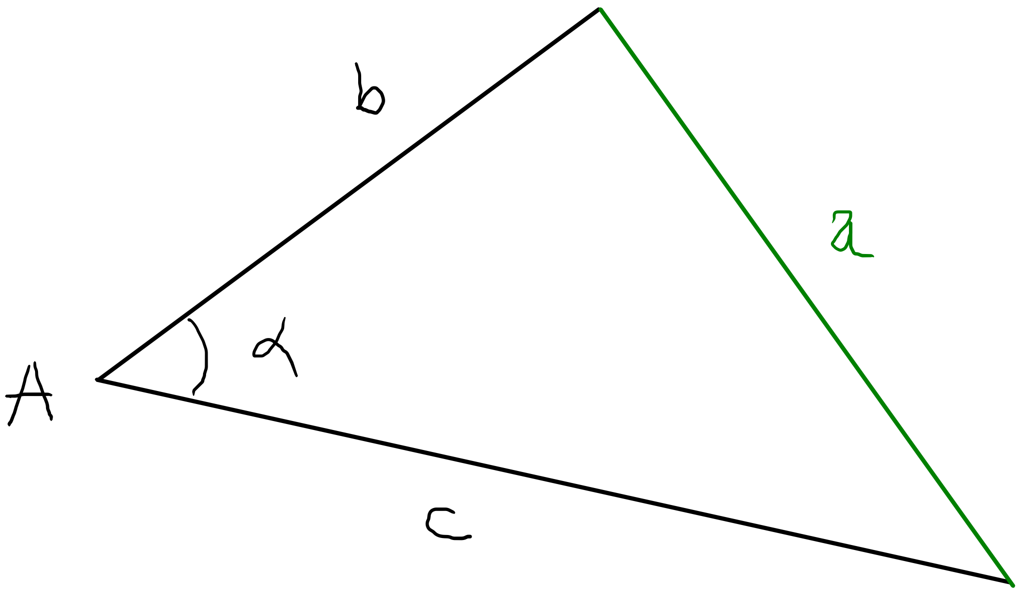
14.04.22

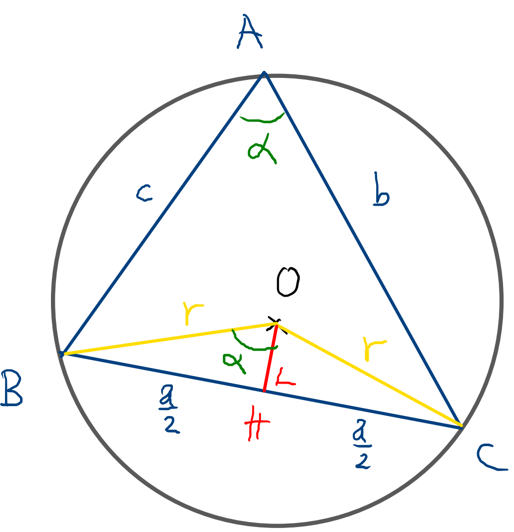
Théorème du cosinus

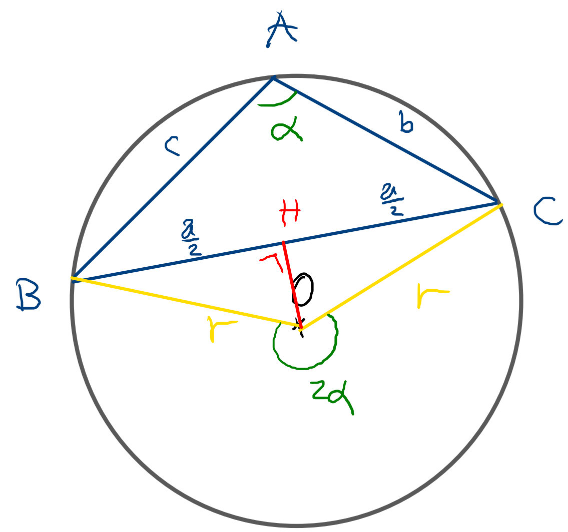


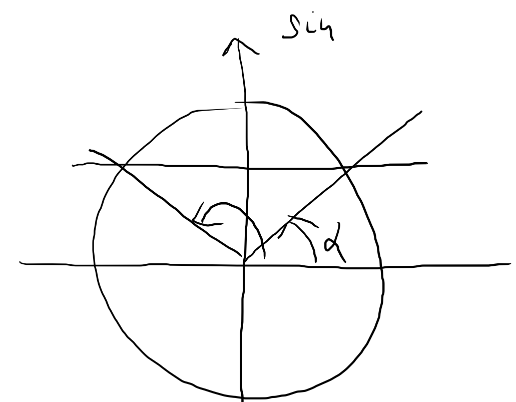
$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

Théorème du sinus

Soit un triangle ABC et son cercle circonscrit de centre O et de rayon r







$\sin(\alpha) = \sin(180° - \alpha)$

$\widehat{BOC} = 2\,\widehat{BAC} = 2\alpha$

$\Rightarrow \widehat{BOH} = \alpha$

$\sin(\alpha) = \dfrac{\frac{a}{2}}{r} = \dfrac{a}{2r}$

$2r = \dfrac{a}{\sin(\alpha)}$

$\widehat{BOC} = 360° - 2\alpha$

$\widehat{BOH} = \frac{1}{2}\widehat{BOC} = 180° - \alpha$

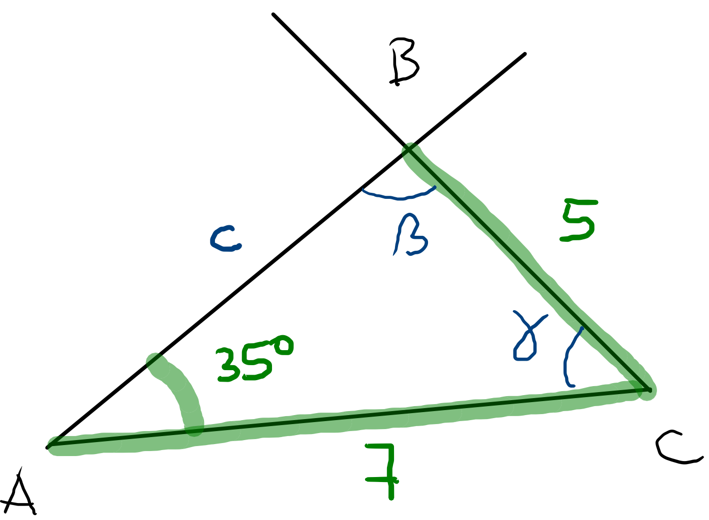
$\underbrace{\sin(180° - \alpha)}_{\sin(\alpha)} = \dfrac{\frac{a}{2}}{r} = \dfrac{a}{2r}$

$\sin(\alpha) = \dfrac{a}{2r} \Rightarrow 2r = \dfrac{a}{\sin(\alpha)}$

On a le théorème du sinus :

$$\boxed{2r = \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}}$$

Appliquons ce résultat



Utilisons le théorème du sinus pour calculer β :

$$\frac{5}{\sin(35°)} = \frac{7}{\sin(\beta)}$$

$$\Rightarrow \quad 5 \cdot \sin(\beta) = 7 \cdot \sin(35°)$$

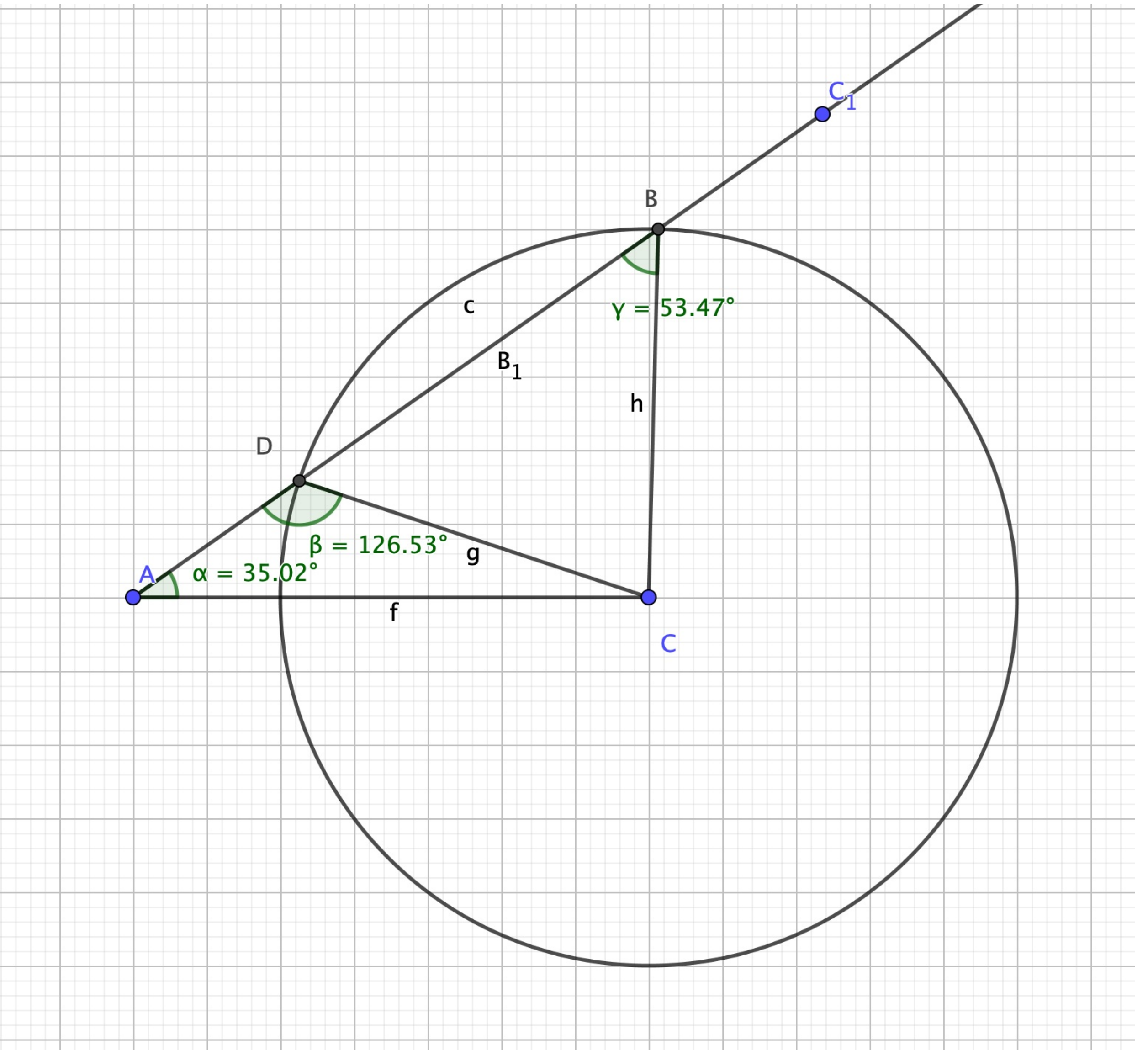
$$\sin(\beta) = \frac{7 \cdot \sin(35°)}{5}$$

$$\sin(\beta) \cong 0.803007010891465 \quad \Rightarrow \boxed{TI} \quad \beta = 53{,}42°$$

Il y a une deuxième solution !

$$\beta \cong \begin{cases} 53{,}42° \\ 180° - 53{,}42° = 126{,}58° \end{cases}$$

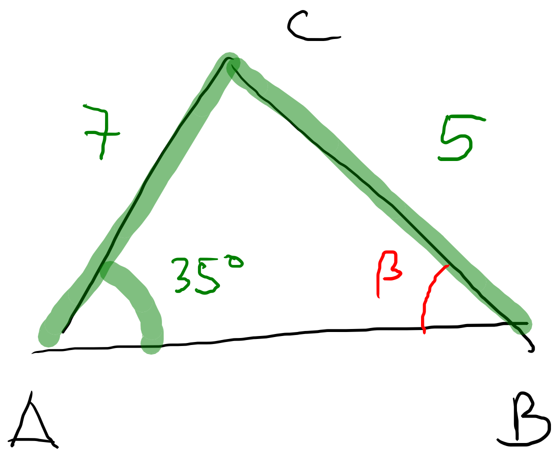
Il y a deux triangles constructibles avec ces valeurs.



4, 4, 4 $\rightarrow$ d)

4, 4, 5

	a	b	c	α	β	γ	A
b)	5	7		35	2 val.		



Cas 1

Cas 2