10.04.25

4.3.4 Résoudre les équations suivantes en donnant les solutions en radians.

e) $\sin(2t) = \cos\left(3t + \frac{\pi}{4}\right)$

f) $\sin\left(\frac{4t}{3}\right) + \cos\left(\frac{t}{2}\right) = 0$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

e) $\underbrace{\sin(2t)}_{\cos\left(\frac{\pi}{2} - 2t\right)} = \cos\left(3t + \frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{2} - 2t\right) = \cos\left(3t + \frac{\pi}{4}\right)$

1) $\frac{\pi}{2} - 2t = 3t + \frac{\pi}{4} + k \cdot 2\pi \quad | \cdot 4$

$2\pi - 8t = 12t + \pi + k \cdot 8\pi \quad | -12t - 2\pi$

$-20t = -\pi + k \cdot 8\pi \quad | \div(-20)$

$t = \frac{\pi}{20} + k \cdot \frac{2\pi}{5}$

2) $\frac{\pi}{2} - 2t = -\left(3t + \frac{\pi}{4}\right) + k \cdot 2\pi \quad | \cdot 4$

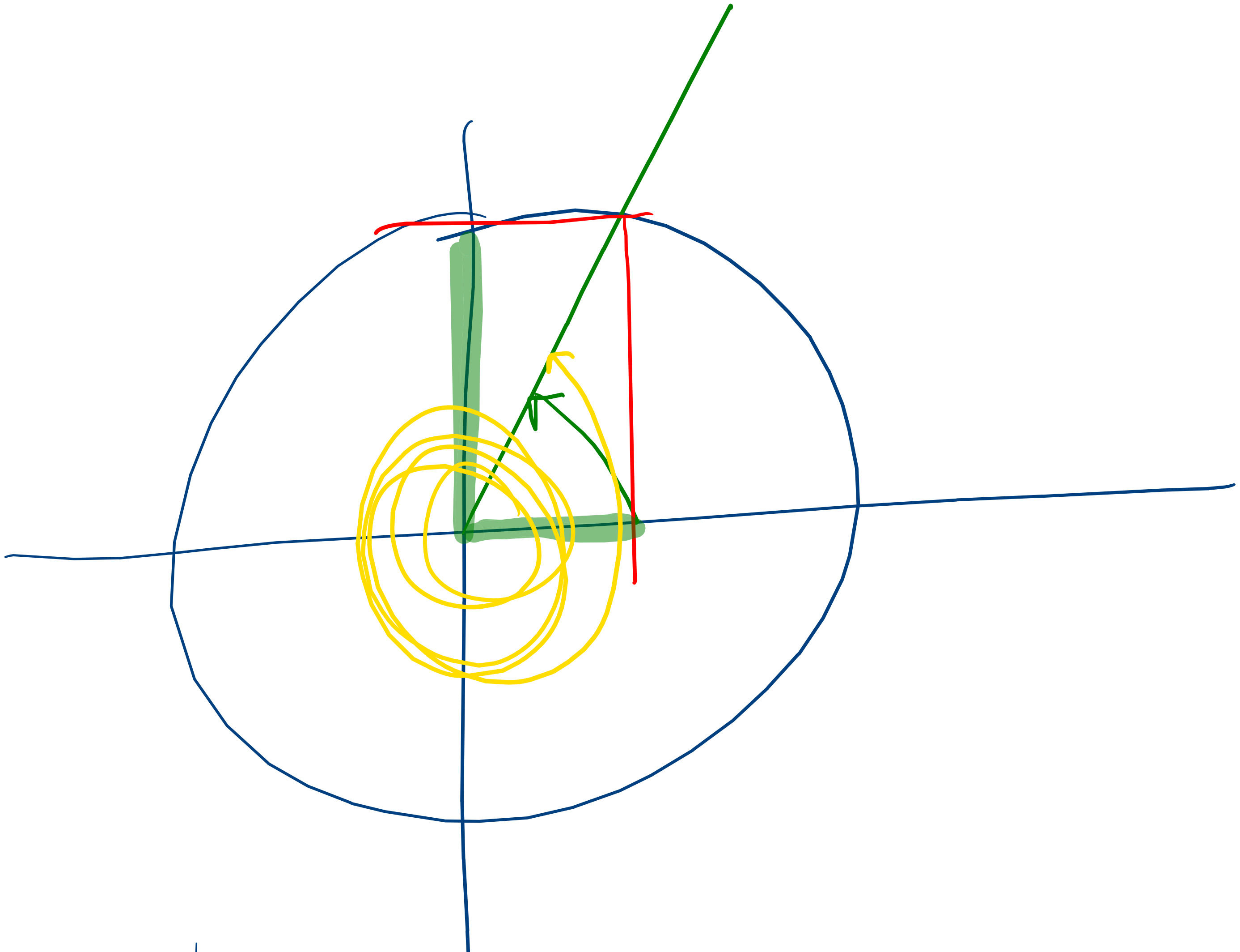
$2\pi - 8t = -12t - \pi + k \cdot 8\pi \quad | +12t - 2\pi$

$4t = -3\pi + k \cdot 8\pi \quad | \div 4$

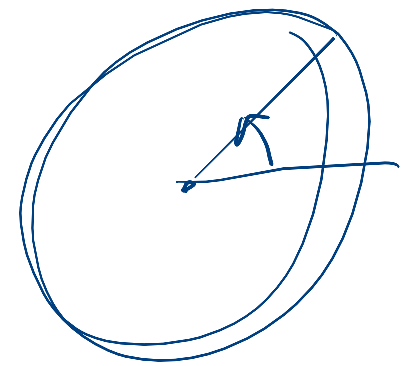
$t = -\frac{3\pi}{4} + k \cdot 2\pi$

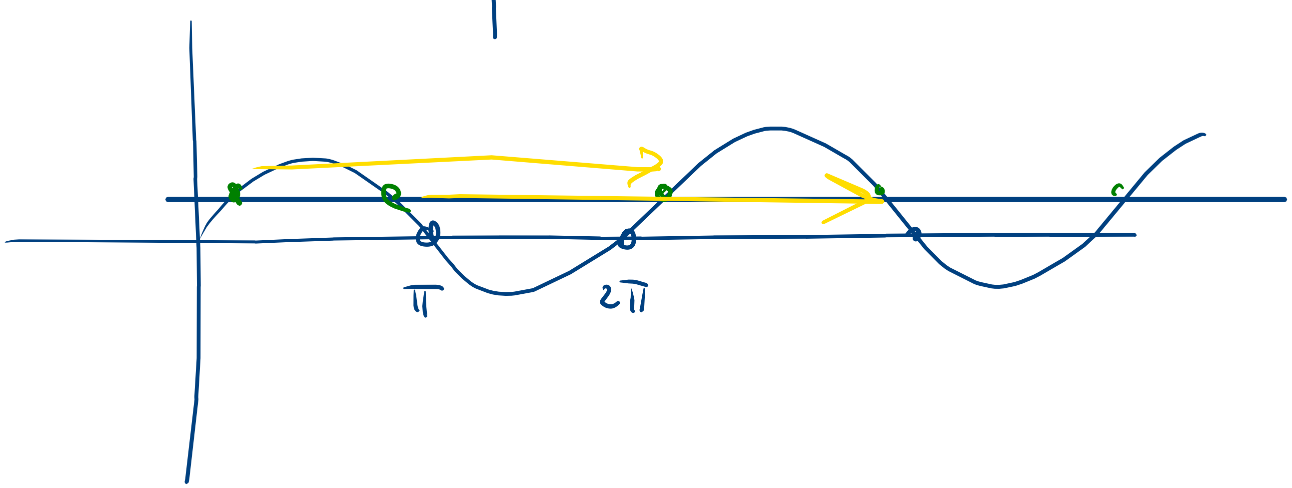
Solutions :

$$t = \begin{cases} \frac{\pi}{20} + k \cdot \frac{2\pi}{5} \\ -\frac{3\pi}{4} + k \cdot 2\pi \end{cases} \quad k \in \mathbb{Z}$$



$$60^\circ + 3600^\circ = 3660^\circ = \alpha$$





f) $\sin\left(\frac{4t}{3}\right) + \cos\left(\frac{t}{2}\right) = 0$

$$\sin\left(\frac{4t}{3}\right) = -\cos\left(\frac{t}{2}\right)$$

$$\Big\downarrow \; *$$

$$\sin\left(\frac{4t}{3}\right) = \cos\left(\pi + \frac{t}{2}\right)$$

$$\Big\downarrow \; **$$

$$\cos\left(\frac{\pi}{2} - \frac{4t}{3}\right) = \cos\left(\pi + \frac{t}{2}\right)$$

	$\cos(\pi - \alpha) = -\cos(\alpha)$
*	$\cos(\pi + \alpha) = -\cos(\alpha)$

**	$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

f) $\sin\left(\frac{4t}{3}\right) + \cos\left(\frac{t}{2}\right) = 0$

$$\sin\left(\frac{4t}{3}\right) = -\cos\left(\frac{t}{2}\right)$$

$*$

$$\sin\left(\frac{4t}{3}\right) = \cos\left(\pi - \frac{t}{2}\right)$$

$$\cos\left(\frac{\pi}{2} - \frac{4t}{3}\right) = \cos\left(\pi - \frac{t}{2}\right)$$

$*$	$\cos(\pi - \alpha) = -\cos(\alpha)$
	$\cos(\pi + \alpha) = -\cos(\alpha)$

$**$	$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

1 $\frac{\pi}{2} - \frac{4t}{3} = \pi - \frac{t}{2} + 2k\pi$

$$3\pi - 8t = 6\pi - 3t + 12k\pi$$

$$-5t = 3\pi + 12k\pi$$

$$t = -\frac{3}{5}\pi + k\frac{12}{5}\pi$$

2 $-\left(\frac{\pi}{2} - \frac{4t}{3}\right) = \pi - \frac{t}{2} + 2k\pi$

$$-\frac{\pi}{2} + \frac{4t}{3} = \pi - \frac{t}{2} + 2k\pi \quad \Big| \cdot 6$$

$$-3\pi + 8t = 6\pi - 3t + 12k\pi$$

$$11t = 9\pi + 12k\pi$$

$$t = \frac{9}{11}\pi + \frac{12}{11}k\pi$$

$$t = \frac{-3}{11}\pi + \frac{12}{11}k\pi$$

f) $\sin\left(\frac{4t}{3}\right) + \cos\left(\frac{t}{2}\right) = 0$

$$-\sin\left(\frac{4t}{3}\right) = \cos\left(\frac{t}{2}\right)$$

$$\cos\left(\frac{\pi}{2} + \frac{4t}{3}\right) = \cos\left(\frac{t}{2}\right)$$

$*$ $\boxed{\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin(\alpha)}$

1) $\frac{\pi}{2} + \frac{4t}{3} = \frac{t}{2} + 2k\pi$

...

$$t = \frac{-3\pi}{5} + k \cdot \frac{12\pi}{5}$$

$$\frac{\pi}{2} + \frac{4t}{3} = -\frac{t}{2} + 2k\pi$$

$$3\pi + 8t = -3t + 12k\pi$$

$$11t = -3\pi + 12k\pi$$

$$t = \frac{-3\pi}{11} + \frac{12}{11}k\pi$$

4.3.5 Résoudre les équations suivantes.

a) $4\cos^2(t) - 4\cos(t) - 3 = 0$

Effectuons le changement de variable $\cos(t) = y$, $y \in [-1;1]$

$$4y^2 - 4y - 3 = 0$$

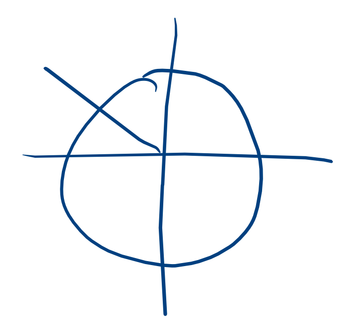
$$\Delta = 16 + 48 = 64 = 8^2$$

- $y_1 = \dfrac{4+8}{8} = \dfrac{12}{8} = \dfrac{3}{2}$ impossible

- $y_2 = \dfrac{4-8}{8} = \dfrac{-4}{8} = -\dfrac{1}{2}$

$\underline{y_1 = \frac{3}{2}}$: impossible

$\underline{y_2 = -\frac{1}{2}}$: $\cos(t) = -\dfrac{1}{2} \overset{TI}{\Rightarrow} t = 120°$



$$t = \begin{cases} 120° + k \cdot 360° \\ -120 + k \cdot 360° \end{cases}$$

b) $2\sin^2(x) - 3\sin(x) + 1 = 0$

Posons $y = \sin(x)$. On résout l'équation :

$$2y^2 - 3y + 1 = 0$$

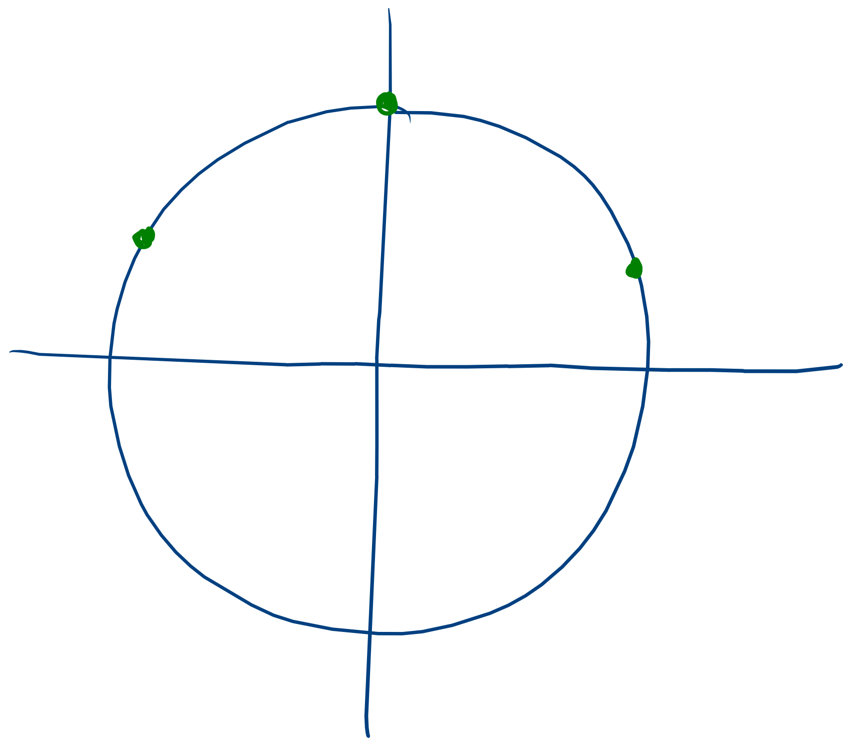
$$(2y - 1)(y - 1) = 0$$

1) $y_1 = \frac{1}{2} \Rightarrow \sin(x) = \frac{1}{2} \overset{\boxed{TI}}{\Rightarrow} x = 30°$

$$\boxed{x = \begin{cases} 30° + k \cdot 360° \\ 150° + k \cdot 360° \end{cases}} \quad k \in \mathbb{Z}$$

2) $y_2 = 1 \Rightarrow \sin(x) = 1 \overset{\boxed{TI}}{\Rightarrow} x = 90°$

$$\boxed{x = 90° + k\,360°} \quad k \in \mathbb{Z}$$



c) $3\sin^2(z) + 8\cos(z) + 1 = 0$

$\sin^2(z) + \cos^2(z) = 1 \Rightarrow \sin^2(z) = 1 - \cos^2(z)$

$3\left(1-\cos^2(z)\right) + 8\cos(z) + 1 = 0$

$3 - 3\cos^2(z) + 8\cos(z) + 1 = 0$

$-3\cos^2(z) + 8\cos(z) + 4 = 0$

$3\cos^2(z) - 8\cos(z) - 4 = 0$

On pose $y = \cos(z)$:

$3y^2 - 8y - 4 = 0$

$\Delta = 64 - 4\cdot 3\cdot(-4) = 64 + 48 = 112$

- $y_1 = \dfrac{8+\sqrt{112}}{6} > 1$ impossible
- $y_2 = \dfrac{8-\sqrt{112}}{6} \simeq -0{,}43050 \Rightarrow \cos(z) = -0{,}43050 \overset{\boxed{T1}}{\Rightarrow} z = 115{,}50^\circ$

$$\boxed{z = \pm 115{,}50^\circ + k\cdot 360^\circ}$$