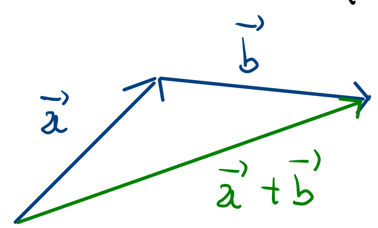
15,01.24

c) On donne les vecteurs $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$ et $\vec{c} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$. Calculer

$\|\vec{a}\| + \|\vec{b}\| + \|\vec{c}\|$; $\|\vec{a} + \vec{b} + \vec{c}\|$; $\|-2\vec{a}\| + 2\|\vec{a}\|$; $\|\vec{a}\|\vec{c}$; $\frac{1}{\|\vec{a}\|}\vec{a}$; $\left\|\frac{1}{\|\vec{a}\|}\vec{a}\right\|$

$$\|\vec{a}\| + \|\vec{b}\| + \|\vec{c}\| = 5 + 13 + 6 = 24$$

$$\|\vec{a} + \vec{b} + \vec{c}\| = \left\|\begin{pmatrix} 9 \\ -1 \end{pmatrix}\right\| = \sqrt{81+1} = \sqrt{82}$$

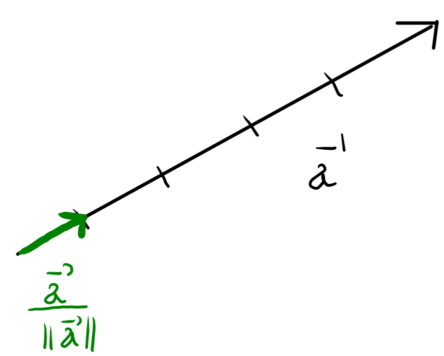


$$\boxed{\|\vec{a}+\vec{b}\| \leqslant \|\vec{a}\| + \|\vec{b}\|}$$ Inégalité triangulaire

$$\|-2\vec{a}\| + 2\|\vec{a}\| = |-2|\|\vec{a}\| + 2\|\vec{a}\| = 2\|\vec{a}\| + 2\|\vec{a}\| = 4\|\vec{a}\| = 20$$

$$\|\vec{a}\|\vec{c} = 5\begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -30 \\ 0 \end{pmatrix}$$

$$\frac{1}{\|\vec{a}\|}\vec{a} = \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$$



$$\boxed{\frac{1}{\|\vec{a}\|}\vec{a} \text{ est toujours unitaire (si } \vec{a} \neq \vec{0})}$$

$$\left\|\frac{1}{\|\vec{a}\|}\vec{a}\right\| = \frac{1}{\|\vec{a}\|}\|\vec{a}\| = 1$$

Propriétés de la norme

1) $\|\vec{a}\| \geq 0$

2) $\|\vec{a}\| = 0 \iff \vec{a} = \vec{0}$

3) $\|\lambda \cdot \vec{a}\| = |\lambda| \cdot \|\vec{a}\|$, $\lambda \in \mathbb{R}$

4) $\frac{1}{\|\vec{a}\|} \cdot \vec{a}$ est un vecteur unitaire $(\vec{a} \neq \vec{0})$

5) $\|\vec{a} + \vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$

d) On donne $\vec{d} = \begin{pmatrix} 8 \\ k-1 \end{pmatrix}$. Calculer le nombre k sachant que la norme de $\vec{d}$ vaut 10.

$$\|\vec{d}\| = 10 \Rightarrow \sqrt{8^2 + (k-1)^2} = 10 \qquad \Big|\ (\ )^2 \qquad ⚠$$

$$64 + k^2 - 2k + 1 = 100$$

$$k^2 - 2k - 35 = 0$$

$$(k-7)(k+5) = 0$$

$k = 7$ ou $k = -5$

10.

e) On donne $\vec{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ et $\vec{v} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. Calculer le nombre m tel que

$$\|\vec{u} + m\vec{v}\| = \sqrt{82}$$

$$\vec{u} + m\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -2m \\ 4m \end{pmatrix} = \begin{pmatrix} 2-2m \\ 3+4m \end{pmatrix}$$

$$\sqrt{(2-2m)^2 + (3+4m)^2} = \sqrt{82} \quad \Big|\ (\ )^2$$

$$4 - 8m + 4m^2 + 9 + 24m + 16m^2 = 82$$

$$20m^2 + 16m - 69 = 0$$

$$\Delta = 16^2 - 4 \cdot 20 \cdot (-69) = 256 + 80 \cdot 69 = 5776 = 76^2$$

$$m_1 = \frac{-16+76}{40} = \frac{60}{40} = \frac{3}{2}$$

$$m_2 = \frac{-16-76}{40} = \frac{-92}{40} = \frac{-23}{10}$$

$$m = \frac{3}{2} \text{ ou } m = \frac{-23}{10}$$

$$S = \left\{ -\frac{23}{10} ; \frac{3}{2} \right\}$$