4.3.6

a) $3\cos(x) + 2\sin(x) = -3$

$$\sin(x) = \frac{-3}{2}\left(1 + \cos(x)\right)$$

Substituons cette expression dans $\sin^2(x) + \cos^2(x) = 1$

$$\frac{9}{4}\left(1 + 2\cos(x) + \cos^2(x)\right) + \cos^2(x) - 1 = 0$$

$$\frac{13}{4}\cos^2(x) + \frac{9}{2}\cos(x) + \frac{5}{4} = 0 \qquad | \cdot 4$$

$$13\cos^2(x) + 18\cos(x) + 5 = 0$$

$$(\cos(x) + 1)(13\cos(x) + 5) = 0$$

(i) $\cos(x) = -1 \iff x = 180° + k \cdot 360°, \quad k \in \mathbb{Z}$

(ii) $\cos(x) = \frac{-5}{13} \implies x = \pm 112{,}62° + k \cdot 360°$

Solutions:

(i) $\cos(x) = -1 \implies \sin(x) = 0 \implies x = 180° + k \cdot 360°$

(ii) $\cos(x) = -\frac{5}{13} \implies \sin(x) = \frac{-3}{2}\left(\frac{8}{13}\right) = \frac{-12}{13} \implies x = -112{,}62° + k \cdot 3$

4.3.6

b) $\sin(t) + 3\cos(t) = 3$

$\sin(t) = -3\cos(t) + 3$

Substituons cette expression dans $\sin^2(t) + \cos^2(t) = 1$

$9\cos^2(t) - 18\cos(t) + 9 + \cos^2(t) - 1 = 0$

$10\cos^2(t) - 18\cos(t) + 8 = 0$

$5\cos^2(t) - 9\cos(t) + 4 = 0$

$(\cos(t) - 1)(5\cos(t) - 4) = 0$

1) $\cos(t) = 1 \Rightarrow \sin(t) = 0 \Rightarrow t = k \cdot 360°$

2) $\cos(t) = \frac{+4}{5} \Rightarrow \sin(t) = \frac{-12}{5} + 3 = \frac{3}{5} \Rightarrow t \cong 36{,}87° + k \cdot 360°$

4.3.6

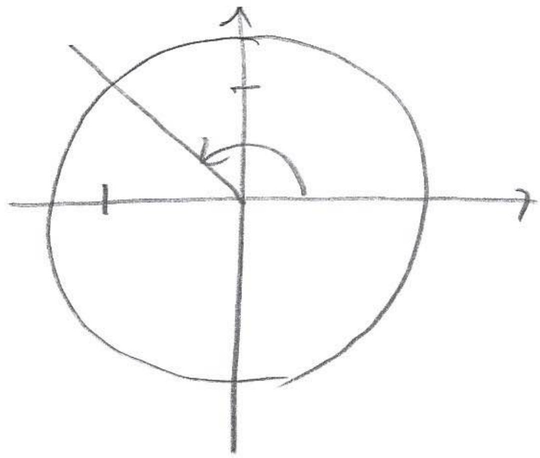
c) $\begin{cases} \sin(x) - \cos(x) = \sqrt{2} \\ \sin^2(x) + \cos^2(x) = 1 \end{cases}$ $\Rightarrow$ $\sin(x) = \cos(x) + \sqrt{2}$

$\cos^2(x) + 2\sqrt{2}\cos(x) + 2 + \cos^2(x) - 1 = 0$

$2\cos^2(x) + 2\sqrt{2}\cos(x) + 1 = 0$. $\quad (\sqrt{2}\cos(x)+1)^2 = 0$

$\Delta = 8 - 8 = 0 \quad \Rightarrow \quad \cos(x) = \frac{-2\sqrt{2}}{4} = -\frac{\sqrt{2}}{2}$

$\cos(x) = -\frac{\sqrt{2}}{2} \quad \Rightarrow \quad \sin(x) = \frac{-\sqrt{2}}{2} + \sqrt{2} = \frac{\sqrt{2}}{2}$



$x = 135° + k \cdot 360°$

4.3.6

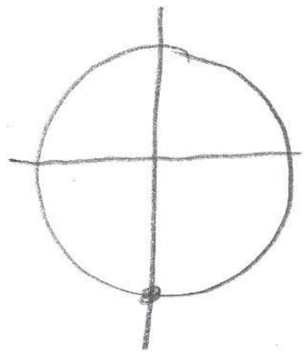
d) $\begin{cases} \sqrt{3}\cos(t) - 1 = \sin(t) \\ \sin^2(t) + \cos^2(t) = 1 \end{cases}$

$$3\cos^2(t) - 2\sqrt{3}\cos(t) + 1 + \cos^2(t) - 1 = 0$$

$$4\cos^2(t) - 2\sqrt{3}\cos(t) = 0$$

$$2\cos^2(t) - \sqrt{3}\cos(t) = 0$$

$$\cos(t)\left(2\cos(t) - \sqrt{3}\right) = 0$$



1) $\cos(t) = 0 \Rightarrow \sin(t) = -1$

$t = -90° + k \cdot 360°$

2) $\cos(t) = \frac{\sqrt{3}}{2} \Rightarrow \sin(t) = \sqrt{3} \cdot \frac{\sqrt{3}}{2} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow t = 30° + k \cdot 360°$

4.3.6

e) $\begin{cases} 3\sin(t) + 5\cos(t) = 2 \\ \sin^2(t) + \cos^2(t) = 1 \end{cases}$ $\Rightarrow$ $\sin(t) = -\frac{5}{3}\cos(t) + \frac{2}{3}$

$$\frac{25}{9}\cos^2(t) - \frac{20}{9}\cos(t) + \frac{4}{9} + \cos^2(t) - 1 = 0 \quad \Big| \cdot 9$$

$$25\cos^2(t) - 20\cos(t) + 4 + 9\cos^2(t) - 9 = 0$$

$$34\cos^2(t) - 20\cos(t) - 5 = 0$$

$\Delta = 1080$

$$\cos(t) = \frac{20 \pm 6\sqrt{30}}{68} = \frac{10 \pm 3\sqrt{30}}{34} = \begin{cases} -0{,}18917 \\ 0{,}77740 \end{cases}$$

1) $\cos(t) \cong -0{,}18917 \Rightarrow \sin(t) \cong 0{,}9815 \Rightarrow t \cong 100{,}90° + k \cdot 360°$

2) $\cos(t) \cong 0{,}77740 \Rightarrow \sin(t) \cong -0{,}629 \Rightarrow t = -38{,}98°$

4.3.6

f) $\begin{cases} \sin(2x) + 3\cos(2x) = 2 \\ \sin^2(2x) + \cos^2(2x) = 1 \end{cases} \Rightarrow \sin(2x) = 2 - 3\cos(2x)$

$$4 - 12\cos(2x) + 9\cos^2(2x) + \cos^2(2x) - 1 = 0$$

$$10\cos^2(2x) - 12\cos(2x) + 3 = 0$$

$$\Delta = 24$$

$$\cos(2x) = \frac{12 \pm 2\sqrt{6}}{20} = \frac{6 \pm \sqrt{6}}{10} \cong \begin{cases} 0{,}35505 \\ 0{,}84495 \end{cases}$$

1) $\cos(2x) \cong 0{,}35505 \Rightarrow \sin(2x) = 0{,}93485$

$$\Rightarrow 2x = 69{,}18^\circ + k \cdot 360^\circ$$

$$x = 34{,}59^\circ + k \cdot 180^\circ$$

2) $\cos(2x) \cong 0{,}84495 \Rightarrow \sin(2x) \cong -0{,}53485$

$$\Rightarrow 2x = -32{,}33^\circ + k \cdot 360^\circ$$

$$x = -16{,}17^\circ + k \cdot 180^\circ$$