2.4.1 Rendre les fractions rationnelles irréductibles :

a) $\dfrac{54a^3b^3}{15a^5b^2}$ b) $\dfrac{-16u^2v^2w^3}{-4u^3vw^2}$ c) $\dfrac{x-1}{2x-2}$

a) $$\frac{54a^3b^3}{15a^5b^2} = \frac{\cancel{3a^3b^2}\cdot 18b}{\cancel{3a^3b^2}\cdot 5a^2} = \frac{18b}{5a^2}$$

b) $$\frac{\overset{4}{\cancel{-16}}\;\overset{1}{\cancel{u^2}}\;\overset{v}{\cancel{v^2}}\;\overset{w}{\cancel{w^3}}}{\underset{1}{\cancel{-4}}\;\underset{u}{\cancel{u^3}}\;\underset{1}{\cancel{v}}\;\underset{1}{\cancel{w^2}}} = \frac{4vw}{u} = \frac{4vw}{u}$$

c) $$\frac{\cancel{x-1}}{2\cancel{(x-1)}} = \frac{1}{2}$$

g) $$\frac{x-x^3}{x^4+2x^3+x^2} = \frac{x(1-x^2)}{x^2(x^2+2x+1)} = \frac{\overset{1}{\cancel{x}}(1-x)\cancel{(1+x)}}{\underset{x}{\cancel{x^2}}(x+1)^{\cancel{2}\,1}}$$

$$= \frac{1-x}{x(x+1)}$$

o) $\frac{2x^3+9x^2+7x-6}{2x^3+x^2-13x+6} = \frac{(x+2)\cancel{(2x-1)}\cancel{(x+3)}}{(x-2)\cancel{(2x-1)}\cancel{(x+3)}} = \frac{x+2}{x-2}$

Posons $p = 2x^3+9x^2+7x-6$ et $q = 2x^3+x^2-13x+6$.

Factorisons p et q.

1 $p = 2x^3+9x^2+7x-6$

$p(1) \neq 0$

$p(-1) = -2+9-7-6 \neq 0$

$p(2) \neq 0$

$p(-2) = -16+36-14-6 = 0 \Rightarrow (x+2)/p$

Par Horner :

	2	9	7	-6
-2		-4	-10	6
	2	5	-3	0

$p = (x+2)(2x^2+5x-3) = (x+2)(2x-1)(x+3)$

2 $q = 2x^3+x^2-13x+6$

$q(1) = 2+1-13+6 \neq 0$

$q(-2) = -16+4+26+6 \neq 0$

$q(-3) = -54+9+39+6 = 0 \Rightarrow (x+3)/q$

Par Horner

	2	1	-13	6
-3		-6	15	-6
	2	-5	2	0

$q = (x+3)(2x^2-5x+2) = (x+3)(2x-1)(x-2)$