2M1 – 6 Avril 2023 Nom & prénom : ....

Analyse VII – Dérivée 2

Exercice 1

Soit la fonction $f(x) = x^2 - 5\,x + 1$.
Calculer $f'(-3)$ en utilisant la définition du calcul d'un nombre dérivé.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{ou} \quad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f(-3) = 9 + 15 + 1 = 25$

$$f'(-3) = \lim_{x \to -3} \frac{f(x) - f(-3)}{x+3}$$

$$= \lim_{x \to -3} \frac{x^2 - 5x + 1 - 25}{x+3} = \lim_{x \to -3} \frac{x^2 - 5x - 24}{x+3}$$

$$= \lim_{x \to -3} \frac{\cancel{(x+3)}(x-8)}{\cancel{x+3}} = -3 - 8 = -11$$

Donc $\underline{f'(-3) = -11}$

Exercice 2

Calculer la dérivée des fonctions suivantes.

a) $f(x) = 3x^2 - 12\,x + 8$

b) $f(x) = x^3 - 3x^2 - 45x + 9$

c) $f(x) = x - 5\,x^2 - \dfrac{16x^5}{5}$

d) $f(x) = 2x\,(x-5) + x^{10} - 4\,x^2 + 450$

a) $f'(x) = 6x - 12$

b) $f'(x) = 3x^2 - 6x - 45$

c) $f'(x) = 1 - 10x - 16x^4$

d) $f(x) = 2x^2 - 10x + x^{10} - 4x^2 + 450$

$= x^{10} - 2x^2 - 10x + 450$

$f'(x) = 10x^9 - 4x - 10$

Exercice 3

Calculer la dérivée des fonctions suivantes.

a) $f(x) = \dfrac{x+1}{x-2}$

c) $f(x) = \dfrac{-3\,x+3}{x^2}$

b) $f(x) = \dfrac{4\,x+9}{-5\,x+4}$

d) $f(x) = \dfrac{3\,x^2+6\,x}{x^2-9}$

a) $u = x+1 \;;\; u' = 1$

$v = x-2 \;;\; v' = 1$

$$f'(x) = \frac{1(x-2) - 1(x+1)}{(x-2)^2} = \frac{x-2-x-1}{(x-2)^2} = \frac{-3}{(x-2)^2}$$

b) $u = 4x+9 \;;\; u' = 4$

$v = -5x+4 \;;\; v' = -5$

$$f'(x) = \frac{4(-5x+4) - (-5)(4x+9)}{(-5x+4)^2}$$

$$= \frac{-20x+16+20x+45}{(-5x+4)^2} = \frac{61}{(-5x+4)^2}$$

c) $u = -3x+3 \;;\; u' = -3$

$v = x^2 \;;\; v' = 2x$

$$f'(x) = \frac{-3 \cdot x^2 - 2x(-3x+3)}{x^4} = \frac{-3x^2+6x^2-6x}{x^4}$$

$$= \frac{3x^2-6x}{x^4} = \frac{3x-6}{x^3}$$

d) $u = 3x^2 + 6x \; ; \quad u' = 6x + 6$

$v = x^2 - 9 \quad ; \quad v' = 2x$

$$f'(x) = \frac{(6x+6)(x^2-9) - 2x(3x^2+6x)}{(x^2-9)^2}$$

$$= \frac{6x^3 + 6x^2 - 54x - 54 - 6x^3 - 12x^2}{(x^2-9)^2}$$

$$= \frac{-6x^2 - 54x - 54}{(x^2-9)^2}$$

Exercice 4

Calculer la dérivée des fonctions suivantes.

a) $f(x) = (2x-6)^3\,(x^2-3x+1)^4$

b) $f(x) = \dfrac{(2x-5)^3}{(x^2+1)^3}$

a) $u = (2x-6)^3$; $u' = 3(2x-6)^2 \cdot 2 = 6(2x-6)^2$

$v = (x^2-3x+1)^4$; $v' = 4(x^2-3x+1)^3 \cdot (2x-3)$

$$f'(x) = 6(2x-6)^2(x^2-3x+1)^4 + (2x-6)^3 \cdot 4(x^2-3x+1)^3(2x-3)$$

$$= 2(2x-6)^2(x^2-3x+1)^3\left[3(x^2-3x+1)+2(2x-6)(2x-3)\right]$$

$$= 2(2x-6)^2(x^2-3x+1)^3\left[3x^2-9x+3+8x^2-36x+36\right]$$

$$= 2(2x-6)^2(x^2-3x+1)^3(11x^2-45x+39)$$

b) $u = (2x-5)^3$; $u' = 6(2x-5)^2$

$v = (x^2+1)^3$; $v' = 6x(x^2+1)^2$

$$f'(x) = \frac{6(2x-5)^2(x^2+1)^3 - (2x-5)^3 \cdot 6x(x^2+1)^2}{(x^2+1)^6}$$

$$= \frac{6(2x-5)^2\cancel{(x^2+1)^2}\left[x^2+1-x(2x-5)\right]}{(x^2+1)^{\cancel{6}\,4}}$$

$$= \frac{6(2x-5)^2(-x^2+5x+1)}{(x^2+1)^4}$$