Bugnon août 2020

Problème 1

a) $ED(f) = \mathbb{R} - \{-1\}$

b) zéros de f : $x^3 - 2x^2 = 0 \iff x^2(x-2) = 0$

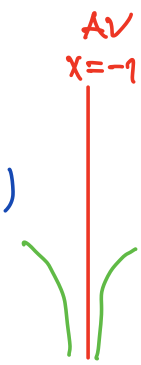
$\iff x = 0$ ou $x = 2$

x		-1		0		2	
$f(x)$	$-$	$\|\|$	$-$	0	$-$	0	$+$
		d		d		s	

c) Recherche des AV :

$$\lim_{x \to -1} f(x) \underset{\text{"}\frac{-3}{0}\text{"}}{=} -\infty \quad \text{(cf tableau des signes)}$$

donc AV : $x = -1$



Recherche AH/AO :

Il y a une AO : $\dfrac{\text{degré } 3}{\text{degré } 2}$

$$\begin{array}{r|l} x^3 - 2x^2 \ \ldots \ \ldots & x^2 + 2x + 1 \\ \hline \underline{x^3 + 2x^2 + x} & x - 4 \\ -4x^2 - x & \\ \underline{-4x^2 - 8x - 4} & \\ 7x + 4 & \end{array}$$

$7x + 4 = \delta(x)$ $\qquad \delta(x) = 0 \iff x = \frac{-4}{7}$

$\boxed{AO : y = x - 4}$

$$f(x) = x - 4 + \frac{7x+4}{(x+1)^2}$$

Position :

x		-1		$-\frac{4}{7}$	
$\delta(x)$	$-$	‖	$-$	0	$+$
Position	dessous	‖	dessous	coupe	dessus

d) $u = x^3 - 2x^2$; $u' = 3x^2 - 4x =$

$v = (x+1)^2$; $v' = 2(x+1)$

$$f'(x) = \frac{(3x^2-4x)\cdot(x+1)^2 - (x^3-2x^2)\cdot 2(x+1)}{(x+1)^4}$$

$$= \frac{\cancel{(x+1)}\left[(3x^2-4x)(x+1) - 2(x^3-2x^2)\right]}{(x+1)^{\cancel{4}3}}$$

$$= \frac{3x^3+3x^2-4x^2-4x-2x^3+4x^2}{(x+1)^3} = \frac{x^3+3x^2-4x}{(x+1)^3}$$

e) zéros de $f'(x)$: $x(x^2+3x-4) = 0$

$x(x+4)(x-1) = 0$

$\downarrow$ 0 $\downarrow$ -4 $\downarrow$ 1

croissance de f :

x		-4		-1		0		1	
$f'(x)$	$+$	0	$-$	‖	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	$\boxed{\text{max}}$	$\searrow$	‖	$\nearrow$	$\boxed{\text{max}}$	$\searrow$	$\boxed{\text{min}}$	$\nearrow$

max $\left(-4; \frac{-32}{3}\right)$ min $\left(1; -\frac{1}{4}\right)$

max $(0; 0)$