2.7.15

$$\boxed{(\sqrt{u})' = \frac{u'}{2\sqrt{u}}}$$

$$\sqrt[n]{u} = u^{\frac{1}{n}} \qquad n \in \mathbb{N}^*$$

$$(\sqrt[n]{u})' = \left(u^{\frac{1}{n}}\right)' = \frac{1}{n} \cdot u^{\frac{1}{n}-1} \cdot u' = \frac{1}{n}\, u^{\frac{1-n}{n}} \cdot u'$$

$$\boxed{(\sqrt[n]{u})' = \frac{u'}{n\sqrt[n]{u^{n-1}}}}$$

a) $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$

b) $f(x) = x^{\frac{1}{5}}$

$$f'(x) = \frac{1}{5} x^{-\frac{4}{5}} = \frac{1}{5\sqrt[5]{x^4}}$$

c) $f(x) = x^{\frac{4}{7}}$

$$f'(x) = \frac{4}{7} x^{-\frac{3}{7}} = \frac{4}{7\sqrt[7]{x^3}}$$

d) $f'(x) = \dfrac{16x - 5}{2\sqrt{8x^2-5x+3}}$

e) $f'(x) = \dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+4}} = \dfrac{x}{\sqrt{x^2+4}}$

f) $f'(x) = \dfrac{3(4x^2-2x)^2(8x-2)}{2\sqrt{(4x^2-2x)^3}}$

$= \dfrac{3(4x^2-2x)^{\cancel{2}} \cdot \cancel{2}(4x-1)}{\cancel{2}\cancel{(4x^2-2x)}\sqrt{4x^2-2x}} = \dfrac{3(\overset{\sqrt{4x^2-2}}{\cancel{4x^2-2}})(4x-1)}{\cancel{\sqrt{4x^2-2}}}$

$= 3\sqrt{4x^2-2x}\,(4x-1)$

g) –

h) –

i) $f(x) = \dfrac{1}{\sqrt{x}} = x^{-1/2}$

$f'(x) = -\dfrac{1}{2}x^{-3/2} = \dfrac{-1}{2\sqrt{x^3}}$

j) –

k) $u = 1+x \quad ; \quad u' = 1$

$v' = \sqrt{1-x} \quad ; \quad v' = \dfrac{-1}{2\sqrt{1-x}}$

$$f'(x) = 1\cdot\sqrt{1-x} + (1+x)\cdot\frac{-1}{2\sqrt{1-x}}$$

$$= \frac{\sqrt{1-x}\cdot 2\sqrt{1-x} - (1+x)}{2\sqrt{1-x}} = \frac{2(1-x)-1-x}{2\sqrt{1-x}}$$

$$= \frac{1-3x}{2\sqrt{1-x}}$$

l) $u = \frac{3x-2}{x+1}$; $u' = \frac{3(x+1)-(3x-2)\cdot 1}{(x+1)^2} = \frac{5}{(x+1)^2}$

$$f'(x) = \frac{\frac{5}{(x+1)^2}}{2\sqrt{\frac{3x-2}{x+1}}} = \frac{5}{2(x+1)^2}\cdot\sqrt{\frac{x+1}{3x-2}} = \frac{5}{2\sqrt{(x+1)^3}\sqrt{3x-2}}$$