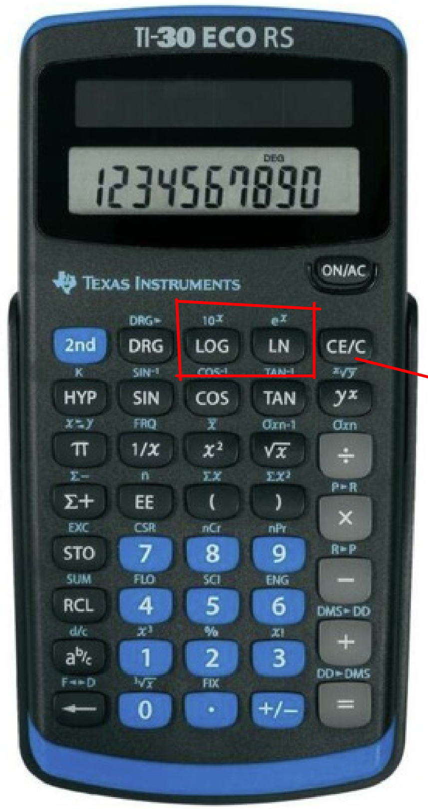
02.09.22

Les logarithmes

Log de base 10 $\quad 10^x = 8 \Leftrightarrow x = \log_{10}(8)$

TI : $\log_{10}(8) = 0.903089986991944$



$10^{0.903089986991944} = 8$

Soit $a \in \mathbb{R}_+^* - \{1\}$

$$\boxed{a^x = y \iff \log_a(y) = x}$$

$\log_{10}(10) = 1$

$\log_{10}(1000) = 3$

$\log_{10}(1) = 0$

$\log_{10}(0,1) = -1$

$\log_3(9) = 2$

$\log_2(1024) = 10$

On note $\log_{10}(x) = \log(x)$

Il existe une autre base privilégiée,

c'est la base $e \cong$ 2.718281828459045

$$\log_e(x) = \ln(x)$$

1.2.2 Calculer à la main :

a) $\log_3(1)$ b) $\log_2(8)$ c) $\log_2(64)$ d) $\log_2(1'024)$

e) $\log_5(5)$ f) $\log_3(\sqrt{3})$ g) $\log_{243}(1/243)$ h) $\log_3(27)$

i) $\log(1'000)$ j) $\log_4(\sqrt{2})$ k) $\log_{1/8}(64)$ l) $\log_5(0,04)$

m) $\log_3(\sqrt[4]{27})$ n) $\ln(e^2)$ o) $\log_a(a)$ p) $\log_a(a^3)$

q) $\log(10000)$ r) $\ln(e)$ s) $\log_2(1/8)$ t) $\log_3(\sqrt[4]{3})$

u) $\log(200) - \log(2)$ v) $\log_6(4) + \log_6(9)$ w) $\log_5(1)$ x) $\log(-1)$

y) $\log(0.0001)$ z) $\ln(0)$

f) $\log_3(\sqrt{3})$ g) $\log_{243}(1/243)$ h) $\log_3(27)$

f) $\log_3(\sqrt{3}) = n \iff 3^n = \sqrt{3}$

$\iff 3^n = 3^{\frac{1}{2}}$

g) $\log_{243}\left(\frac{1}{243}\right) = n \iff 243^n = \frac{1}{243}$

$\iff 243^n = 243^{-1}$

j) $\log_4(\sqrt{2}) = n \iff 4^n = \sqrt{2}$

$4^n = \sqrt{\sqrt{4}}$

$4^n = \left(4^{1/2}\right)^{1/2}$

$4^n = 4^{1/4}$

j) $\log_4(\sqrt{2}) = n \iff 4^n = 2^{1/2}$

$2^{2n} = 2^{1/2}$

$\Rightarrow n = \frac{1}{4}$

Propriétés de log

Soit $a \in \mathbb{R}_+^* - \{1\}$. Soit u, v, x et y tels que

$a^x = u$ et $a^y = v$.

On a $\log_a(u) = x$ et $\log_a(v) = y$.

1 $$\log_a(\underbrace{a^x \cdot a^y}_{a^{x+y}}) = \log_a(a^{x+y}) = x + y$$

$$= \log_a(a^x) + \log_a(a^y)$$

$$\boxed{\log_a(u \cdot v) = \log_a(u) + \log_a(v)}$$

2 $$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$$

3 $$\log_a(u^n) = n \cdot \log_a(u)$$

Mercredi 1.2.2