09.11.22

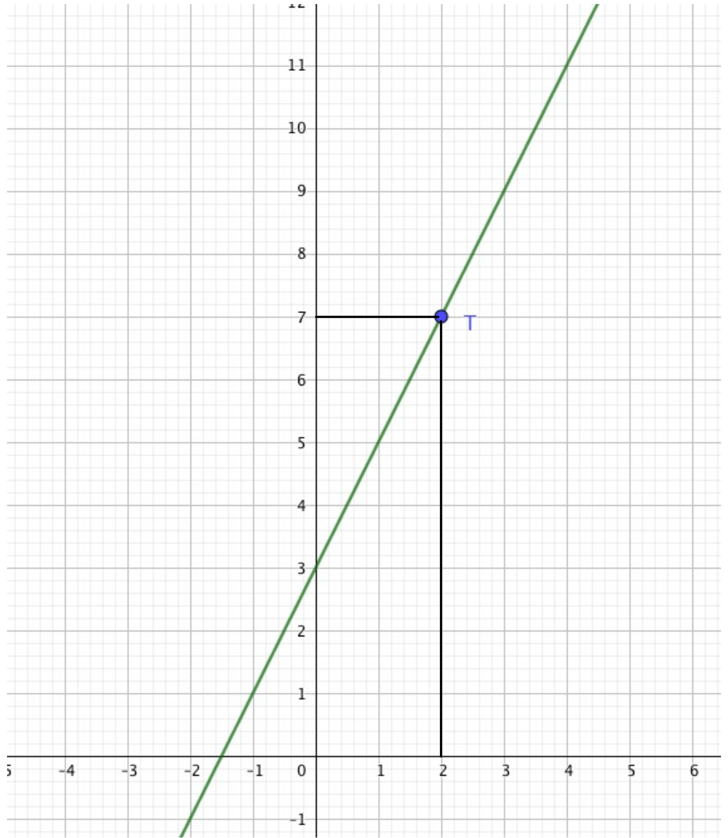
Limites

Nous avons vu que

$$f(x) = \frac{2x^2 - x - 6}{x - 2} \qquad ED(f) = \mathbb{R} - \{2\}$$

$$\lim_{x \to 2} f(x) = 7.$$

On dit que $f(x)$ présente un point trou en $x = 2$. Ses coordonnées sont $T(2;7)$



Soit $g(x) = \dfrac{2x^2 - x - 5}{x - 2}$

$ED(g) = \mathbb{R} - \{2\}$

Que se passe-t-il pour $g(x)$ si x tend vers 2 ?

$g(1{,}9) = -3{,}2$

$g(1{,}99) \cong -93{,}02$

$g(1{,}999) \cong -993{,}002$

$g(2{,}1) \cong 17{,}2$

$g(2{,}01) \cong 107{,}02$

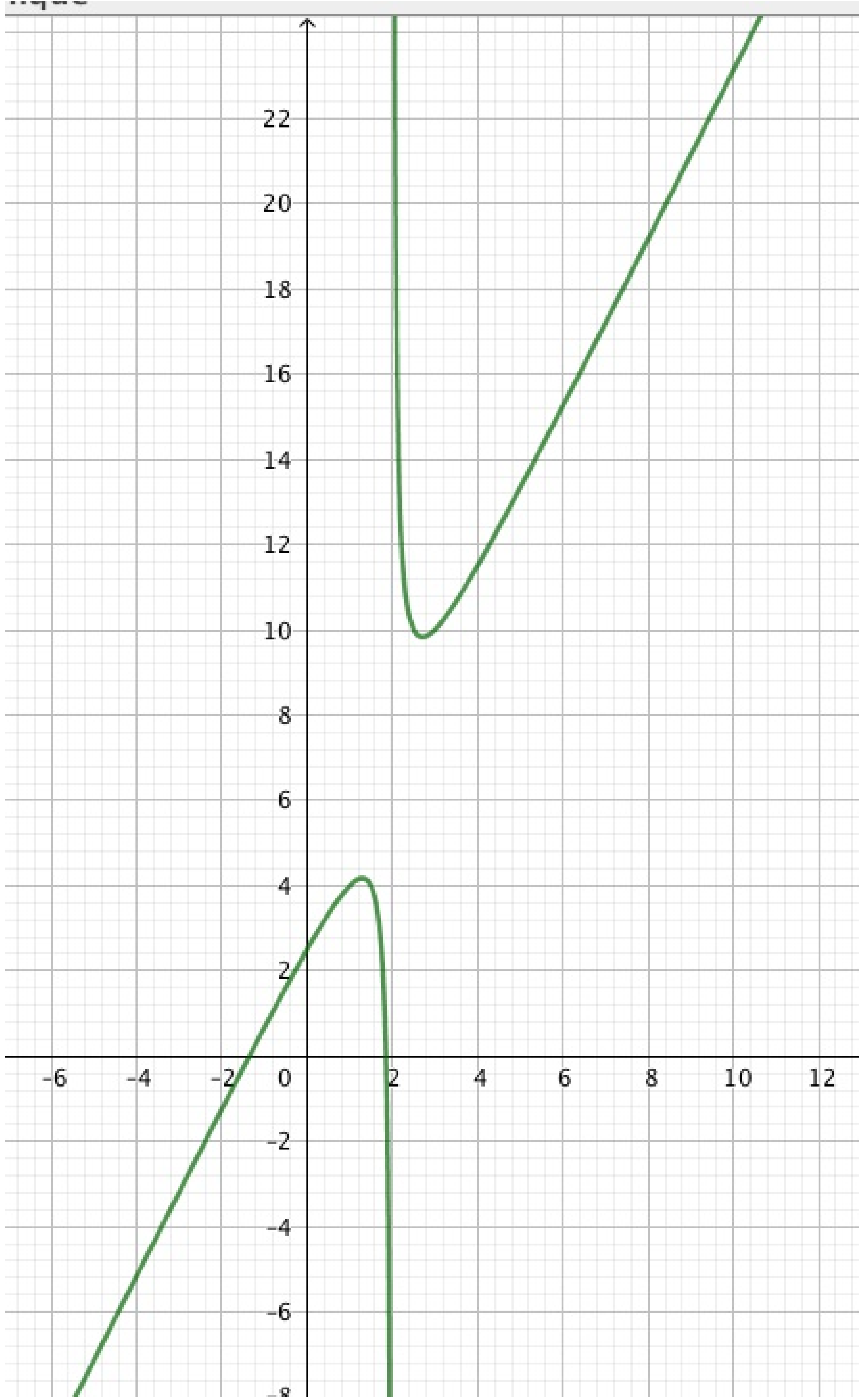
$g(2{,}001) = 1007{,}002$

$$\boxed{\lim_{x \to 2} \frac{2x^2 - x - 5}{x - 2} \underset{\text{"}\frac{1}{0}\text{"}}{=} \infty}$$

En fait, on peut écrire

$$\lim_{\substack{x \to 2 \\ <}} g(x) = -\infty$$

$$\lim_{\substack{x \to 2 \\ >}} g(x) = +\infty$$



22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-6
-4
-2
0
2
4
6
8
10
12

2.4.2 Calculer les limites suivantes :

a) $\lim\limits_{x\to 1}(4x^3-2x^2+x-1)$

b) $\lim\limits_{x\to -2}(x^2-5x)$

c) $\lim\limits_{x\to 0}\dfrac{x+3x^2}{x+1}$

d) $\lim\limits_{x\to 4}(-5)$

e) $\lim\limits_{x\to 3}\sqrt{x^2-5}$

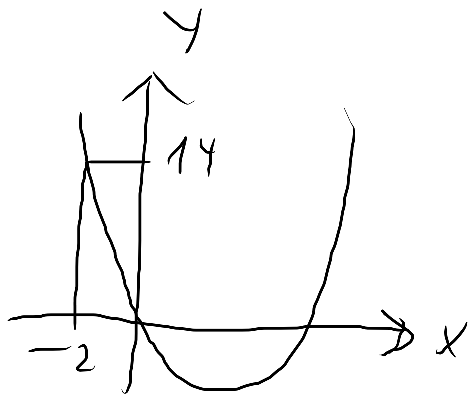
f) $\lim\limits_{x\to 1}\dfrac{x^2+2x+1}{x^3+x^2+x}$

a) $ED(f)=\mathbb{R}$, $f(x)=4x^3-2x^2+x-1$

$$\lim_{x\to 1} f(x) = f(1) = 4-2+1-1 = 2$$

b) $f(x)=x^2-5x$, $ED(f)=\mathbb{R}$

$$\lim_{x\to -2}(x^2-5x) = 4+10 = 14$$



c) $f(x)=\dfrac{x+3x^2}{x+1}$ $\qquad ED(f)=\mathbb{R}-\{-1\}$

$$\lim_{x\to 0} f(x) = \frac{0}{1} = 0$$

e) $\lim_{x \to 3} \sqrt{x^2 - 5}$

$f(x) = \sqrt{x^2 - 5}$

Recherchons $ED(f)$:

x		$-\sqrt{5}$		$\sqrt{5}$	
$x^2 - 5$	$+$	0	$-$	0	$+$

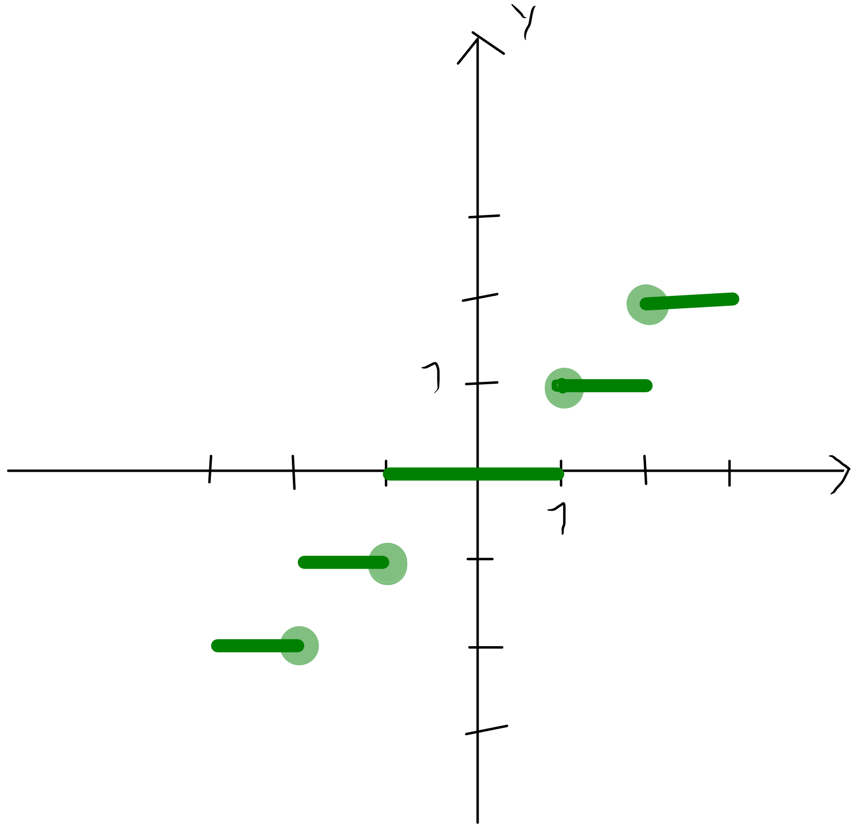
$ED(f) =]-\infty ; -\sqrt{5}] \cup [\sqrt{5} ; +\infty[$

$3 \in ED(f)$: $\lim_{x \to 3} f(x) \sqrt{9-5} = 2$

$$\lim_{\substack{x \to \sqrt{5} \\ >}} \sqrt{x^2 - 5} = 0$$

Un autre exemple de limite

$f(x) = [x]$, $ED(f) = \mathbb{R}$



$\lim\limits_{x \to 0} [x] = 0$

$\lim\limits_{x \to 1} [x]$ n'existe pas !

$\lim\limits_{\substack{x \to 1 \\ <}} [0] = 0$ et $\lim\limits_{\substack{x \to 1 \\ >}} [x] = 1$

2.4.3 Calculer les limites suivantes :

a) $\lim_{x\to 3} \frac{x-3}{2x-6}$

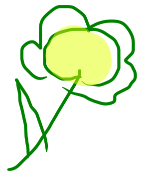
$$\lim_{x\to 3} \frac{x-3}{2x-6} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{ind}}{=}} \lim_{x\to 3} \frac{\overset{1}{\cancel{x-3}}}{2\cancel{(x-3)}} = \frac{1}{2}$$

ind : indéterminé

d) $\lim_{x\to 1} \frac{x^2-1}{x-1} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{ind}}{=}} \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\underset{1}{\cancel{x-1}}} = \lim_{x\to 1} \frac{x+1}{1} = \frac{2}{1}$

$$= 2$$

g) $\lim\limits_{x\to 1} \dfrac{x^2-x}{x^3-4x^2-7x+10} \overset{\text{ind}}{=} \text{"}\dfrac{0}{0}\text{"}$

$$\lim_{x\to 1} \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x^2-3x-10)} =$$ 

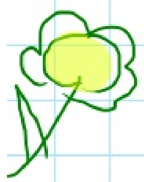
Factorisons $x^3-4x^2-7x+10 = d(x)$ par Horner

$p(1) = 0 \Rightarrow (x-1) / d(x)$

Horner :

	1	−4	−7	10
1		1	−3	−10
	1	−3	−10	0

$$x^3-4x^2-7x+10 = (x-1)(x^2-3x-10)$$

 $$= \lim_{x\to 1} \frac{x}{x^2-3x-10} = \frac{1}{1-3-10} = \frac{1}{-12} = -\frac{1}{12}$$