16.11.22

h) $$\lim_{x\to-2}\frac{x+\sqrt{x+6}}{x+\sqrt{2-x}} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{ind}}{=}} \lim_{x\to-2}\frac{x+\sqrt{x+6}}{\underbrace{x+\sqrt{2-x}}_{A+B}}\cdot\frac{x-\sqrt{2-x}}{\underbrace{x-\sqrt{2-x}}_{A-B}}$$

$$= \lim_{x\to-2}\frac{\left(x+\sqrt{x+6}\right)\left(x-\sqrt{2-x}\right)}{\underbrace{x^2-(2-x)}_{A^2-B^2}}$$

$$= \lim_{x\to-2}\frac{\left(x+\sqrt{x+6}\right)\left(x-\sqrt{2-x}\right)}{x^2+x-2}$$

$$\underset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to-2}\frac{\left(x+\sqrt{x+6}\right)\left(x-\sqrt{2-x}\right)}{(x+2)(x-1)}\cdot\frac{x-\sqrt{x+6}}{x-\sqrt{x+6}}$$

$$= \lim_{x\to-2}\frac{\left(x^2-(x+6)\right)\left(x-\sqrt{2-x}\right)}{(x+2)(x-1)\left(x-\sqrt{x+6}\right)}$$

$$= \lim_{x\to-2}\frac{\left(x^2-x-6\right)\left(x-\sqrt{2-x}\right)}{(x+2)(x-1)\left(x-\sqrt{x+6}\right)}$$

$$= \lim_{x\to-2}\frac{(x-3)\cancel{(x+2)}\left(x-\sqrt{2-x}\right)}{\cancel{(x+2)}(x-1)\left(x-\sqrt{x+6}\right)}$$

$$= \frac{(-5)\cancel{\left(-2-\sqrt{4}\right)}}{(-3)\cancel{\left(-2-\sqrt{4}\right)}} = \frac{5}{3}$$

2.4.5 Calculer, si elles existent, la limite à gauche, la limite à droite et la limite des fonctions suivantes pour x tendant vers x_0 :

a) $f(x) = \frac{|x|}{x}$ $\quad x_0 = 0$ $\qquad$ b) $f(x) = \frac{x^2 + |x|}{|x|}$ $\quad x_0 = 0$

b) $\lim\limits_{x \to 0} \frac{x^2 + |x|}{|x|} \overset{\text{ind}}{=} \text{"}\frac{0}{0}\text{"}$?

$f(4) = \frac{16 + |4|}{|4|} = \frac{16+4}{4} = \frac{20}{4} = 5$

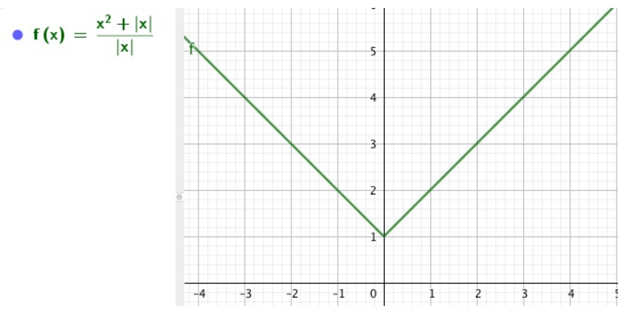
$f(-4) = \frac{16 + |-4|}{|-4|} = \frac{16+4}{4} = 5$

• $\lim\limits_{\substack{x \to 0 \\ >}} \frac{x^2 + x}{x} \overset{\text{ind}}{=} \text{"}\frac{0}{0}\text{"} \lim\limits_{\substack{x \to 0 \\ >}} \frac{\cancel{x}(x+1)}{\cancel{x}_1} = 1$

• $\lim\limits_{\substack{x \to 0 \\ <}} \frac{x^2 - x}{-x} = \lim\limits_{\substack{x \to 0 \\ <}} \frac{\cancel{x}(x-1)}{\cancel{-x}_{-1}} = \frac{-1}{-1} = 1$

Ainsi $\lim\limits_{\substack{x \to 0 \\ <}} f(x) = \lim\limits_{\substack{x \to 0 \\ >}} f(x) = 1$, donc

$$\boxed{\lim_{x \to 0} f(x) = 1}$$



2.4.14

a) $\lim_{x \to -2} \frac{x^2+3x+6}{(x+2)^2} \underset{\text{“}\frac{4}{0_+}\text{”}}{=} +\infty$

on dit que cette limite tend vers l'infini