28.04.23

2.7.19 Former l'équation de la tangente au graphe de f en son point d'abscisse a, si :

a) $f(x) = 1 + 2x - x^3$, $a = 1$

b) $f(x) = \dfrac{x+3}{x}$, $a = 3$

a) Point $T(1\,;\,2)$

La tangente : $y = \underbrace{f'(1)}_{\text{pente de la tangente en } T} \cdot x + h$

- $f'(x) = 2 - 3x^2$
- pente $f'(1) = 2 - 3 = -1$

La tangente : $y = -x + h$

La tangente passe par T : $2 = -1 + h \Rightarrow h = 3$

La tangente est $\boxed{y = -x + 3}$

b) $f(x) = \frac{x+3}{x}$, $a = 3$

- Point $T(3;2)$

- $f'(x) = \frac{x-(x+3)}{x^2} = \frac{x-x-3}{x^2} = \frac{-3}{x^2}$

$u = x+3 \quad ; \quad u' = 1$

$v = x \quad ; \quad v' = 1$

- Pente $f'(3) = \frac{-3}{9} = \frac{-1}{3}$

- Tangente $y = -\frac{1}{3}x + h$

$2 = -\frac{1}{3} \cdot 3 + h$

$2 = -1 + h \Rightarrow h = 3$

$y = -\frac{1}{3}x + 3$

c) $f(x) = \sqrt{2x+1}, a = 4$

Formulaire: $(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$

- Point $T(4; 3)$

- $f'(x) = \dfrac{\cancel{2}^{1}}{\cancel{2}\sqrt{2x+1}} = \dfrac{1}{\sqrt{2x+1}}$

- Pente : $f'(4) = \dfrac{1}{3}$

- Tangente $y = \dfrac{1}{3}x + h$

$$3 = \underbrace{\frac{1}{3} \cdot 4}_{\frac{4}{3}} + h \Rightarrow h = 3 - \frac{4}{3} = \frac{5}{3}$$

$$y = \frac{1}{3}x + \frac{5}{3}$$