

1.1.8

31.08.22

m)  $3\sqrt{5} - 4\sqrt{20} + 5\sqrt{45} - 3\sqrt{80}$

n)  $2\sqrt{40} - 2\sqrt{90} + \sqrt{4'000} - 5\sqrt{10}$

n)  $\sqrt{40} = 2\sqrt{10}$

$\sqrt{90} = 3\sqrt{10}$

$\sqrt{4000} = 2\sqrt{1000} = 2 \cdot 10\sqrt{10} = 20\sqrt{10}$

$= 4\sqrt{10} - 6\sqrt{10} + 20\sqrt{10} - 5\sqrt{10} = 13\sqrt{10}$

**1.1.9** Effectuer et réduire :

a)  $(9\sqrt{12} + 3)(\sqrt{3} + 8)$

$\downarrow$   
 $2\sqrt{3}$

$= (\underline{18\sqrt{3}} + 3)(\underline{\sqrt{3}} + 8)$

$= 54 + 144\sqrt{3} + 3\sqrt{3} + 24$

$= 78 + 147\sqrt{3}$

$18\sqrt{3} \cdot \sqrt{3} = 18 \underbrace{\sqrt{3}\sqrt{3}}_3$

$18 \underbrace{\sqrt{2} \cdot \sqrt{6}}_{\sqrt{12}} = 18\sqrt{12} = 36\sqrt{3}$

## Exposant rationnel

$$1) (3^2)^{\frac{1}{2}} = 3^{2 \cdot \frac{1}{2}} = 3^1 = 3$$

$$2) (8^3)^{\frac{1}{3}} = 8^{3 \cdot \frac{1}{3}} = 8^1 = 8$$

$$1) (3^{\frac{1}{2}})^2 = (\sqrt{3})^2 = 3$$

$$2) (8^{\frac{1}{3}})^3 = (2)^3 = 8$$

$$1) 3^{\frac{1}{2}} = \sqrt{3}$$

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$2) 8^{\frac{1}{3}} = \sqrt[3]{8}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

Formule  $(x^p)^{\frac{1}{q}} = (x^{\frac{1}{q}})^p = x^{\frac{p}{q}} = \sqrt[q]{x^p}$

1.1.10 Simplifier les expressions suivantes :

a)  $\sqrt[3]{\sqrt{7}}$

b)  $\sqrt[3]{2^{18} \cdot 5^{12} \cdot 3^3}$

c)  $\sqrt[4]{64} \cdot \sqrt[4]{4}$

d)  $\sqrt[5]{3^{15}}$

e)  $(\sqrt[8]{\sqrt[4]{\sqrt{2}}})^{128}$

a)  $\sqrt[3]{\sqrt{7}} = \left(7^{\frac{1}{2}}\right)^{\frac{1}{3}} = 7^{\frac{1}{6}} = \sqrt[6]{7}$

$$(x^n)^m = x^{n \cdot m}$$

b)  $\left(2^{18}\right)^{\frac{1}{3}} \cdot \left(5^{12}\right)^{\frac{1}{3}} \cdot \left(3^3\right)^{\frac{1}{3}}$   
 $2^6 \cdot 5^4 \cdot 3$

c)  $\sqrt[4]{64} \cdot \sqrt[4]{4} = \sqrt[4]{64 \cdot 4} = \sqrt[4]{256} = 4$

d)  $\sqrt[5]{3^{15}} = 3^{\frac{15}{5}} = 3^3 = 27$

$$g) \sqrt[3]{5 \sqrt{5 \sqrt{5}}}$$

$$\left( 5 \left( 5 \cdot 5^{\frac{1}{2}} \right)^{\frac{1}{2}} \right)^{\frac{1}{3}}$$
$$\left( 5 \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{1}{4}} \right)^{\frac{1}{3}}$$

f)  $\sqrt{3\sqrt{3}}$    g)  $\sqrt[3]{5\sqrt{5\sqrt{5}}}$    h)  $\sqrt{2\sqrt[3]{2}}$

$$f) \left(3^1 \cdot 3^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(3^{\frac{3}{2}}\right)^{\frac{1}{2}} = 3^{\frac{3}{4}} = \sqrt[4]{3^3} = \sqrt[4]{27}$$

$$g) \sqrt[3]{5\sqrt{5\sqrt{5}}} = \left(5 \left(5^1 \cdot 5^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(5 \cdot \left(5^{\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}$$

$$x^p \cdot x^q = x^{p+q}$$

$$(x^p)^q = x^{p \cdot q}$$

$$= \left(5 \cdot 5^{\frac{3}{4}}\right)^{\frac{1}{3}} = \left(5^{\frac{7}{4}}\right)^{\frac{1}{3}} = 5^{\frac{7}{12}} = \sqrt[12]{5^7}$$

$$h) \sqrt{2\sqrt[3]{2}} = \left(2 \cdot 2^{\frac{1}{3}}\right)^{\frac{1}{2}} = \left(2^{\frac{4}{3}}\right)^{\frac{1}{2}} = 2^{\frac{2}{3}} = \sqrt[3]{4}$$

1.1.11

i)  $\frac{\sqrt[3]{a^4}}{\sqrt{a}}$

j)  $\frac{\sqrt[6]{a^5}}{\sqrt[4]{a^3}}$

k)  $\frac{\sqrt{a} \cdot \sqrt[3]{a}}{\sqrt[4]{a^3}}$

l)  $\frac{a^3}{\sqrt[3]{a^5} \cdot \sqrt[6]{a}}$

i)  $\frac{a^{\frac{4}{3}}}{a^{\frac{1}{2}}} = a^{\frac{4}{3}} \cdot a^{-\frac{1}{2}} = a^{\frac{5}{6}}$

$$\frac{4}{3} - \frac{1}{2} = \frac{8}{6} - \frac{3}{6} = \frac{5}{6}$$

l)  $\frac{a^3}{a^{\frac{5}{3}} \cdot a^{\frac{1}{6}}} = \frac{a^3}{a^{\frac{11}{6}}} = a^3 \cdot a^{-\frac{11}{6}} = a^{\frac{7}{6}}$

$$\frac{5}{3} + \frac{1}{6} = \frac{10}{6} + \frac{1}{6}$$