

Propriétés (suite et fin)

⑧ log ln

On aimerait calculer le \log_a , pour a quelconque ($a > 0, a \neq 1$).

$$\log_a(x) = \log_a\left(b^{\log_b(x)}\right) \stackrel{\textcircled{7}}{=} \log_b(a) \cdot \log_a(b)$$

$\textcircled{7} \log_a(x^r) = r \log_a(x)$

Donc $\textcircled{8} \log_b(x) = \frac{\log_a(x)}{\log_a(b)}$

Par exemple : $\ln(100) \stackrel{\textcircled{11}}{\approx} 4.605170185988091$

$$\ln(100) = \frac{\log_{10}(100)}{\log_{10}(e)} \approx \frac{2}{0.434294481903252} \approx 4.605170185988092$$

$$\log_{27}(42) = \frac{\ln(42)}{\ln(27)} \approx 1,134 \Leftrightarrow 27^{1,134} \approx 42$$

4.2.3 Sachant que $\log(2) = 0.3010$ et $\log(3) = 0.4771$, calculer sans la calculatrice :

a) $\log(6)$ b) $\log(16)$ c) $\log(\sqrt{2})$ d) $\log(0,5)$ e) $\log(36)$ f) $\log\left(\frac{8}{27}\right)$

$$\begin{aligned} \text{a) } \log(6) &= \log(2 \cdot 3) = \log(2) + \log(3) \\ &= 0,3010 + 0,4771 = 0,7781 \end{aligned}$$

$$\text{b) } \log(16) = \log(2^4) = 4 \cdot \log(2) = 1,2040$$

$$\text{c) } \log(\sqrt{2}) = \log\left(2^{\frac{1}{2}}\right) = \frac{1}{2} \log(2) = 0,1505$$

$$\text{d) } \log(0,5) = \log\left(\frac{1}{2}\right) = \log(2^{-1}) = -\log(2) = -0,3010$$

$$\begin{aligned} \text{e) } \log(36) &= \log(4 \cdot 9) = \log(4) + \log(9) = 2 \log(2) + 2 \log(3) \\ &= 1,5562 \end{aligned}$$

$$\begin{aligned} \text{f) } \log\left(\frac{8}{27}\right) &= \log(8) - \log(27) = 3 \log(2) - 3 \log(3) \\ &= 3 (\log(2) - \log(3)) = -0,5283 \end{aligned}$$

4.2.4 Simplifier les expressions ci-dessous sans utiliser la machine :

a) $\log(16) + 2\log(3) - 2\log(2) - \frac{1}{2}\log(9)$ b) $\log(15) + 3\log(10) - \log(30) - \log(5)$

c) $4\log(5) + \log\left(\frac{1}{5}\right) - 3\log(3) + \frac{1}{3}\log(27)$ d) $\frac{\log(20) + \log(100) - \log(2)}{\log(5'000) - \log(5) + \log(0,1)}$

$$\begin{aligned} \text{a) } & 4\log(2) + 2\log(3) - 2\log(2) - \log(3) \\ & = \log(2)(4 - 2) + \log(3)(2 - 1) \end{aligned}$$

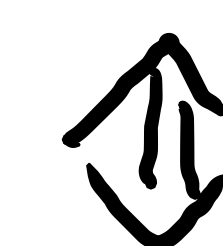
$$= 2\log(2) + \log(3) = \log(12)$$

4.2.5 Résoudre les équations ci-dessous :

a) $x = \log_2(32)$ b) $2^x = 100$ c) $\log_x(256) = 4$ d) $\log_2(x) = 4$

e) $10^x = 5$ f) $e^{2x-1} = 27$ g) $\log_x(1'000) = 3$ h) $12^x = -49$

$$a = b$$



$$\log(a) = \log(b)$$

b) $2^x = 100$

$$x = \log_2(100)$$
$$x = \frac{\log(100)}{\log(2)}$$
$$x = \frac{2}{\log(2)}$$

$2^x = 100$

$$\log(2^x) = \log(100)$$
$$x \cdot \log(2) = 2$$
$$x = \frac{2}{\log(2)}$$

4.2.6 Résoudre les équations ci-dessous :

a) $\log_{11}(x+1) = \log_{11}(7)$ b) $\log_6(2x-3) = \log_6(12) - \log_6(3)$

c) $\log(x) - \log(x+1) = 3 \log(4)$ d) $2 \log_3(x) = 3 \log_3(5)$

e) $\ln(x) + \ln(x-2) = 0,5 \ln(9)$ f) $\log_8(x+4) = 1 - \log_8(x-3)$

c) $\log(x) - \log(x+1) = 3 \log(4)$ (**)

$$\log(x) = \log(x+1) + 3 \log(4)$$

$$\log(x) = \log(x+1) + \log(64)$$

$$\log(x) = \log(64 \cdot (x+1))$$

$$\Rightarrow (*) \quad x = 64(x+1)$$

$$x = 64x + 64$$

$$63x = -64$$

$$x = \frac{-64}{63}$$

est une solution de (*)

Preuve : Comme $\log\left(\frac{-64}{63}\right)$ n'existe pas, donc l'équation (**)
n'a pas de solution