2.10.2

06.03.24

e) $f(x) = \dfrac{(x-1)^2}{x+2}$

$ED(f) = \mathbb{R} - \{-2\}$

Calcul de la dérivée :

$u = (x-1)^2 \quad , \quad u' = 2(x-1)$

$v = x + 2 \quad , \quad v' = 1$

$$f'(x) = \frac{2(x-1)(x+2) - (x-1)^2}{(x+2)^2} = \frac{(x-1)\left[2x+4-x+1\right]}{(x+2)^2}$$

$$= \frac{(x-1)(x+5)}{(x+2)^2}$$

$ED(f') = \mathbb{R} - \{-2\} = ED(f)$

Tableau de la croissance :

x		-5		-2		1	
$f'(x)$	$+$	0	$-$	‖	$-$	0	$+$
$f(x)$	↗	$\boxed{\text{max}}$	↘	‖	↘	$\boxed{\text{min}}$	↗

max $(-5 ; -12)$

min $(1 ; 0)$

$$f(-5) = \frac{(-5-1)^2}{-5+2} = \frac{36}{-3} = -12$$

$$f(1) = \frac{(1-1)}{1+2} = 0$$

g) $f(x) = x^2\sqrt{6 - x^2}$

① Recherchons $ED(f)$. La condition est $6 - x^2 \geq 0$

$$6 - x^2 \geq 0$$
$$(\sqrt{6} - x)(\sqrt{6} + x) \geq 0$$

x		$-\sqrt{6}$		$\sqrt{6}$	
$6-x^2$	$-$	0	$+$	0	$-$

$ED(f) = [-\sqrt{6}\,;\sqrt{6}]$

② Dérivons $f(x)$:

$(uv)' = u'v + uv'$

$u = x^2$; $u' = 2x$

$v = \sqrt{6 - x^2}$; $v' = \dfrac{-\cancel{2}x}{\cancel{2}\sqrt{6-x^2}} = \dfrac{-x}{\sqrt{6-x^2}}$

$$f'(x) = 2x\sqrt{6-x^2} + x^2 \cdot \frac{-x}{\sqrt{6-x^2}}$$

$$= \frac{2x(6-x^2) - x^3}{\sqrt{6-x^2}} = \frac{12x - 3x^3}{\sqrt{6-x^2}} = \frac{3x(4-x^2)}{\sqrt{6-x^2}}$$

$$= \frac{3x(2-x)(2+x)}{\sqrt{6-x^2}}$$

$ED(f') =]-\sqrt{6}\,;\sqrt{6}[$

Tableau de la croissance

x		$-\sqrt{6}$		-2		0		2		$\sqrt{6}$	
$f'(x)$		$\|\|$	$+$	0	$-$	0	$+$	0	$-$	$\|\|$	
$f(x)$		$\boxed{\min}$	$\nearrow$	$\boxed{\max}$	$\searrow$	$\boxed{\min}$	$\nearrow$	$\boxed{\max}$	$\searrow$	$\boxed{\min}$	

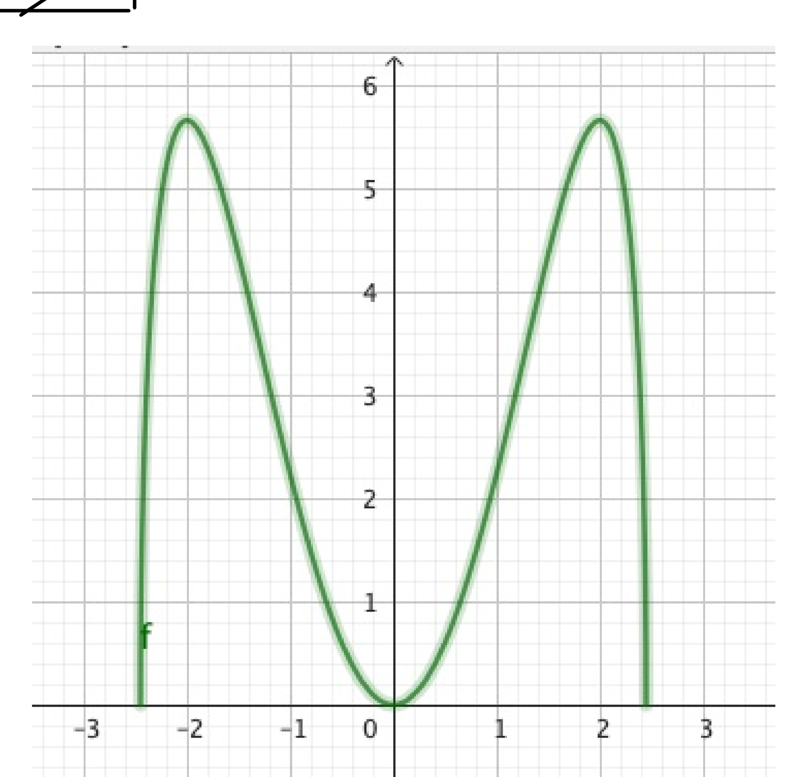
$f(-\sqrt{6}) = 0$ min $(-\sqrt{6}\,; 0)$

$f(-2) = 4 \cdot \sqrt{2}$ max $(-2, 4\sqrt{2})$

$f(0) = 0$ min $(0, 0)$

$f(2) = 4 \cdot \sqrt{2}$ max $(2\,; 4\sqrt{2})$

$f(\sqrt{6}) = 0$ min $(\sqrt{6}\,; 0)$



h) $f(x) = \sin(x)(1+\cos(x))$, sur $[0; 2\pi]$

$ED(f) = \mathbb{R}$

$u = \sin(x) \quad ; \quad u' = \cos(x)$

$v = 1 + \cos(x) \quad ; \quad v' = -\sin(x)$

$$f'(x) = \cos(x)\left[1 + \cos(x)\right] + \sin(x)\cdot(-\sin(x))$$

$$= \cos(x) + \underbrace{\cos^2(x) - \sin^2(x)}$$

CRM

$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = 1 - 2\sin^2(\alpha) = 2\cos^2(\alpha) - 1$

V1 $\quad f'(x) = \cos(x) + \cos(2x)$

$\cos(\pi - \alpha) = -\cos(\alpha)$

zéro : $\quad -\cos(x) = \cos(2x)$

$\cos(\pi - x) = \cos(2x)$

$$\begin{cases} \pi - x = 2x + 2k\pi \\ \pi - x = -2x + 2k\pi \end{cases} \Leftrightarrow \begin{cases} -3x = -\pi + 2k\pi \\ x = -\pi + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{3} + 2k\frac{\pi}{3} \\ x = \pi + 2k\pi \end{cases} \quad k \in \mathbb{Z}$$

Les zéros de $f'(x)$ dans $[0; 2\pi]$: $\frac{\pi}{3}$ $\quad \pi \quad$ $\frac{5\pi}{3}$

$k=0$ $\quad$ $k=0$, $k=1$ $\quad$ $k=2$

	0°		60°		180°		300°		360°
x	0		$\frac{\pi}{3}$		π		$\frac{5\pi}{3}$		2π
$f'(x)$		$+$	0	$-$	0	$-$	0	$+$	
$f(x)$		$\nearrow$	max	$\searrow$		$\searrow$	min	$\nearrow$	

$f(x) = \cos(x) + \cos(2x)$

max $\left(\frac{\pi}{3} ; \frac{3\sqrt{3}}{4}\right)$

min $\left(\frac{5\pi}{3} ; \frac{-3\sqrt{3}}{4}\right)$

$$f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)\left(1 + \cos\left(\frac{\pi}{3}\right)\right) = \frac{\sqrt{3}}{2}\left(1 + \frac{1}{2}\right) = \frac{3\sqrt{3}}{4}$$

$$f\left(\frac{5\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right)\left(1 + \cos\left(\frac{5\pi}{3}\right)\right) = -\frac{\sqrt{3}}{2}\left(1 + \frac{1}{2}\right) = -\frac{3\sqrt{3}}{4}$$

V_2

$$f'(x) = \cos(x) + \cos^2(x) - \sin^2(x)$$

$$= \cos(x) + \cos^2(x) - \left(1 - \cos^2(x)\right)$$

$$= 2\cos^2(x) + \cos(x) - 1$$

$$= \left(2\cos(x) - 1\right)\left(\cos(x) + 1\right)$$

$$\cos(x) = -1 \qquad \Rightarrow \quad x = \pi + 2k\pi$$

$$\cos(x) = \frac{1}{2} \qquad \Rightarrow \quad \begin{cases} x = \frac{\pi}{3} + 2k\pi \\ x = \frac{5\pi}{3} + 2k\pi \end{cases} \qquad \ldots$$