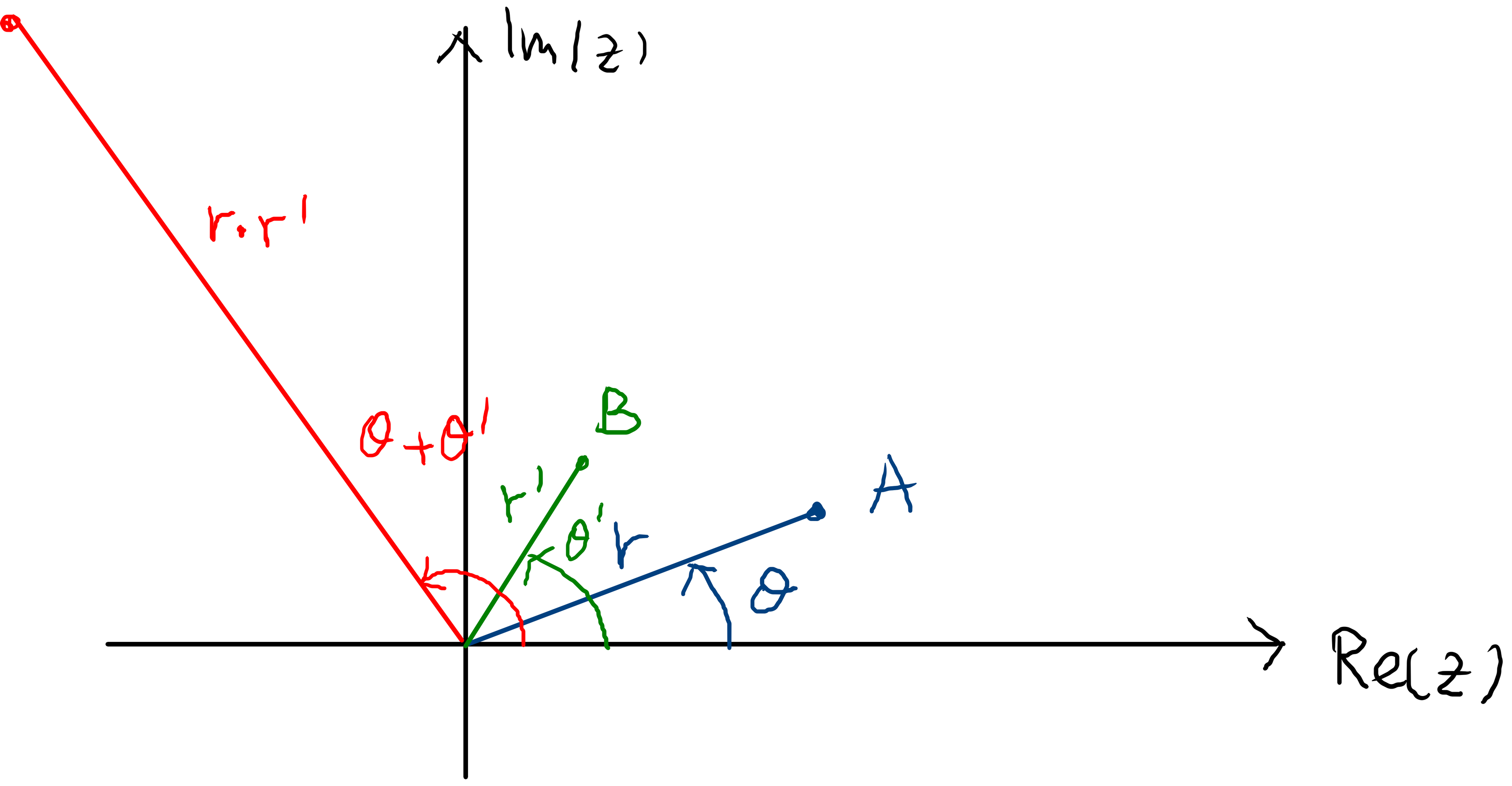
06, 09, 23

Proposition

Soit $z = [r, \theta]$ et $z' = [r', \theta']$ deux nombres complexes



z affixe du point A

z' affixe du point B

$$z \cdot z' = [r \cdot r' ; \theta + \theta']$$

Démonstration :

On a $z = r(\cos\theta + \sin\theta\, i)$ et $z' = r'(\cos\theta' + \sin\theta'\, i)$

Calculons $z \cdot z'$:

$$r(\cos\theta + \sin\theta\, i) \cdot r'(\cos\theta' + \sin\theta'\, i) = rr'\left[\cos\theta\cos\theta' + \sin\theta\sin\theta' i^2 + \sin\theta\cos\theta' i + \cos\theta\sin\theta' i\right]$$

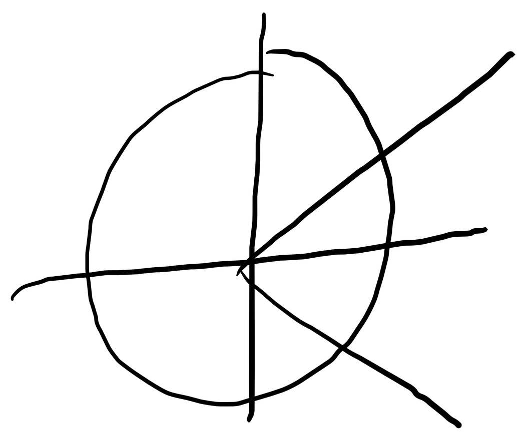
$$= rr'\left[\underbrace{\cos\theta\cos\theta' - \sin\theta\sin\theta'}_{\text{CRM pg 30}} + \underbrace{(\sin\theta\cos\theta' + \cos\theta\sin\theta')}_{\text{CRM pg 30}} i\right]$$

$$= rr'\left[\cos(\theta + \theta') + \sin(\theta + \theta')\, i\right] = [rr' ; \theta + \theta'] \quad \text{cqfd}$$

La notation trigonométrique est très utile pour multiplier deux nombres complexes

Exemple : $[5 ; \frac{\pi}{2}] \cdot [2 ; \frac{\pi}{3}] = [10 ; \frac{5\pi}{6}]$

Proposition



$$\frac{z}{z'} = \frac{[r; \theta]}{[r', \theta']} = \left[\frac{r}{r'} ; \theta - \theta'\right] \quad \text{où} \quad z' \neq 0$$

CRM

$\cos(-\alpha) = \cos\alpha$
$\sin(-\alpha) = -\sin(\alpha)$
$\sin(\alpha) = -\sin(-\alpha)$

Démonstration :

$$\frac{z}{z'} = \frac{r(\cos\theta + \sin\theta\, i)}{r'(\cos\theta' + \sin\theta'\, i)} = \frac{r}{r'} \; \frac{\cos\theta + \sin\theta\, i}{\cos\theta' + \sin\theta'\, i} \cdot \frac{\cos\theta' - \sin\theta'\, i}{\cos\theta' - \sin\theta'\, i}$$

$$= \frac{r}{r'} \; \frac{[1; \theta] \cdot \left(\cos(-\theta') + \sin(-\theta')\, i\right)}{\underbrace{(\cos\theta')^2 + (\sin\theta')^2}_{=1}}$$

$$= \frac{r}{r'} \; [1; \theta] \cdot [1; -\theta'] = \frac{r}{r'} \; [1 ; \theta - \theta'] = \left[\frac{r}{r'}, \theta - \theta'\right] \quad \text{cqfd}$$

Exemple :

$$\left[6; \frac{2\pi}{3}\right] \div \left[3; \frac{-\pi}{3}\right] = \left[2; \frac{2\pi}{3} + \frac{\pi}{3}\right] = [2; \pi]$$

Formule de Moivre

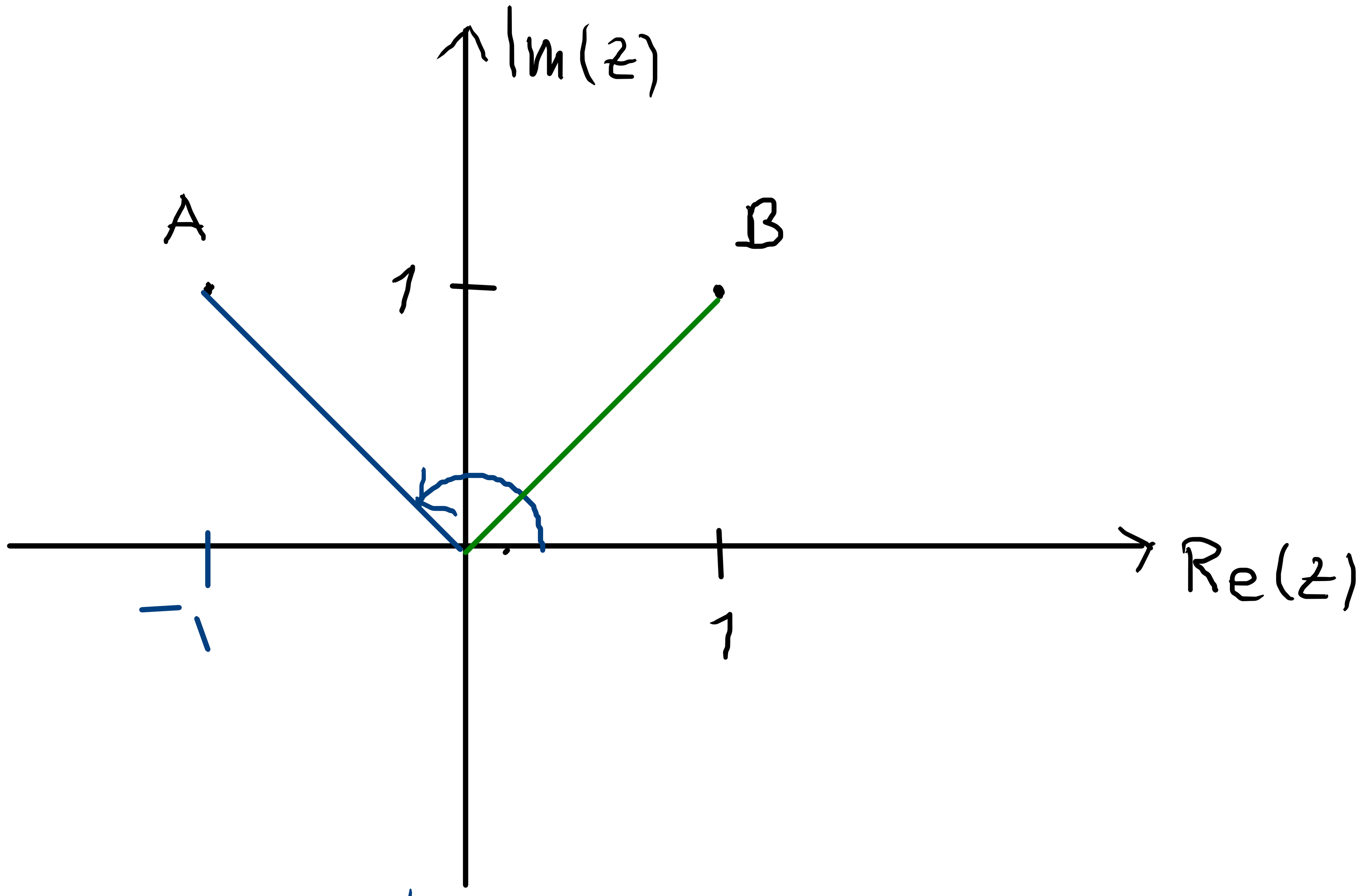
Soit $z = [r, \theta]$, $n \in \mathbb{N}^*$. On a

$$z^n = [r^n ; n\theta]$$

Démonstration : à suivre

1.2.5 Calculer $z_1\, z_2$ et z_1/z_2 en utilisant la forme trigonométrique :

a) $\overset{A}{z_1} = -1 + i, \quad z_2 = \overset{B}{1} + i$



$$\boxed{\begin{aligned} z_1 &= [\sqrt{2}\,;\ 135°] \\ z_2 &= [\sqrt{2}\,;\ 45°] \end{aligned}}$$

- $z_1 \cdot z_2 = [2\,;\ 180°] = -2$
- $\dfrac{z_1}{z_2} = [1\,;\ 90°] = i$

$$z_1 = \sqrt{2}\left(\cos\theta + \sin\theta\, i\right)$$

$$= \sqrt{2}\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\, i\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\, i\right)$$

$$\begin{cases} \cos\theta = -\dfrac{\sqrt{2}}{2} \\ \sin\theta = \dfrac{\sqrt{2}}{2} \end{cases} \Rightarrow \theta = 135°$$