15.09.23

1.3.3 Résoudre dans $\mathbb{C}$ les équations ci-dessous :

a) $z^4 - (6+3i)z^3 + (8+12i)z^2 = 0$

b) $z^3 + 2z^2 + (-4+4i)z + 16 + 16i = 0$, sachant que -4 est un zéro

c) $z^3 - 4z^2 + (8+i)z - 7 + i = 0$, sachant que $1-i$ est un zéro

a) $z^2\left(\underbrace{z^2 - (6+3i)z + (8+12i)}\right) = 0$

$$D = (6+3i)^2 - 4\cdot(8+12i)$$
$$= 36 - 9 + 36i - 32 - 48i = -5 - 12i$$

Il faut trouver $d = a+bi$ tel que $d^2 = D$.

$$\begin{array}{l} \text{Re} \\ \text{module} \\ \text{Im} \end{array} \begin{cases} a^2 - b^2 = -5 \\ a^2 + b^2 = 13 \\ 2ab = -12 \end{cases} \Leftrightarrow \begin{cases} a^2 = 4 \\ b^2 = 9 \\ ab = -6 \end{cases} \Leftrightarrow \begin{cases} a = \pm 2 \\ b = \pm 3 \\ ab = -6 \end{cases}$$

$d_1 = 2 - 3i \; ; \; d_2 = -2 + 3i$

$$z_1 = \frac{6+3i+2-3i}{2} = 4 \; ; \; z_2 = \frac{6+3i-2+3i}{2} = \frac{4+6i}{2} = 2+3i$$

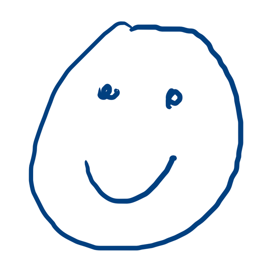
$$S = \{0 ; 4 ; 2+3i\}$$

c) $\underbrace{z^3 - 4z^2 + (8+i)z - 7 + i}_{P(z)} = 0$, sachant que $1 - i$ est un zéro

- $P(1-i) \mid P(z) \Rightarrow (z-(1-i)) \mid P(z)$

Par Horner :

	1	-4	$8+i$	$-7+i$
$1-i$	$\downarrow$	$1-i$	$-4+2i$	$7-i$
	1	$-3-i$	$4+3i$	0



$(-3-i)(1-i) = -4+2i$

$(4+3i)(1-i) = 7-i$

$$(z-(1-i))\left(\underbrace{z^2 - (3+i)z + 4 + 3i}\right) = 0$$

$$D = (3+i)^2 - 4(4+3i)$$
$$= 8 + 6i - 16 - 12i = -8 - 6i$$

On cherche $d = a + bi$ tel que $d^2 = D$

$$\begin{array}{l} \text{Re} \\ \text{module} \\ \text{Im} \end{array} \begin{cases} a^2 - b^2 = -8 \\ a^2 + b^2 = 10 \\ 2ab = -6 \end{cases} \Leftrightarrow \begin{cases} a^2 = 1 \\ b^2 = 9 \\ ab = -3 \end{cases} \Leftrightarrow \begin{cases} a = \pm 1 \\ b = \pm 3 \\ ab = -3 \end{cases}$$

$d_1 = 1 - 3i \quad , \quad d_2 = 1 + 3i$

$z_1 = 1 - i \quad ; \quad z_2 = \dfrac{3+i+1-3i}{2} = \dfrac{4-2i}{2} = 2 - i$

$z_3 = \dfrac{3+i+1+3i}{2} = \dfrac{4+4i}{2} = 2 + 2i$

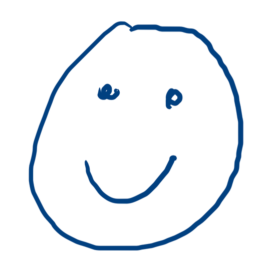
$S = \{ 1-i \; ; \; 2-i \; ; \; 2+2i \}$

c) $\underbrace{z^3 - 4z^2 + (8+i)z - 7 + i}_{P(z)} = 0$, sachant que $1-i$ est un zéro

- $P(1-i) \mid P(z) \quad \Rightarrow \quad (z-(1-i)) \mid P(z)$

Par Horner :

	1	-4	$8+i$	$-7+i$
$1-i$	$\downarrow$	$1-i$	$-4+2i$	$7-i$
	1	$-3-i$	$4+3i$	0



$(-3-i)(1-i) = -4+2i$

$(4+3i)(1-i) = 7-i$

$$(z-(1-i))\left(\underbrace{z^2 - (3+i)z + 4 + 3i}\right) = 0$$

$$D = (3+i)^2 - 4(4+3i)$$
$$= 8+6i - 16 - 12i = -8-6i$$

On cherche $d = a+bi$ tel que $d^2 = D$

$$\begin{array}{l} \text{Re} \\ \text{module} \\ \text{Im} \end{array} \begin{cases} a^2 - b^2 = -8 \\ a^2 + b^2 = 10 \\ 2ab = -6 \end{cases} \Leftrightarrow \begin{cases} a^2 = 1 \\ b^2 = 9 \\ ab = -3 \end{cases} \Leftrightarrow \begin{cases} a = \pm 1 \\ b = \pm 3 \\ ab = -3 \end{cases}$$

$d_1 = 1-3i \quad , \quad d_2 = -1+3i$

$z_1 = 1-i \quad ; \quad z_2 = \dfrac{3+i+1-3i}{2} = \dfrac{4-2i}{2} = 2-i$

$$z_3 = \frac{3+i-1+3i}{2} = \frac{2+4i}{2} = 1+2i$$

$S = \{1-i \;;\; 2-i \;;\; 1+2i\}$