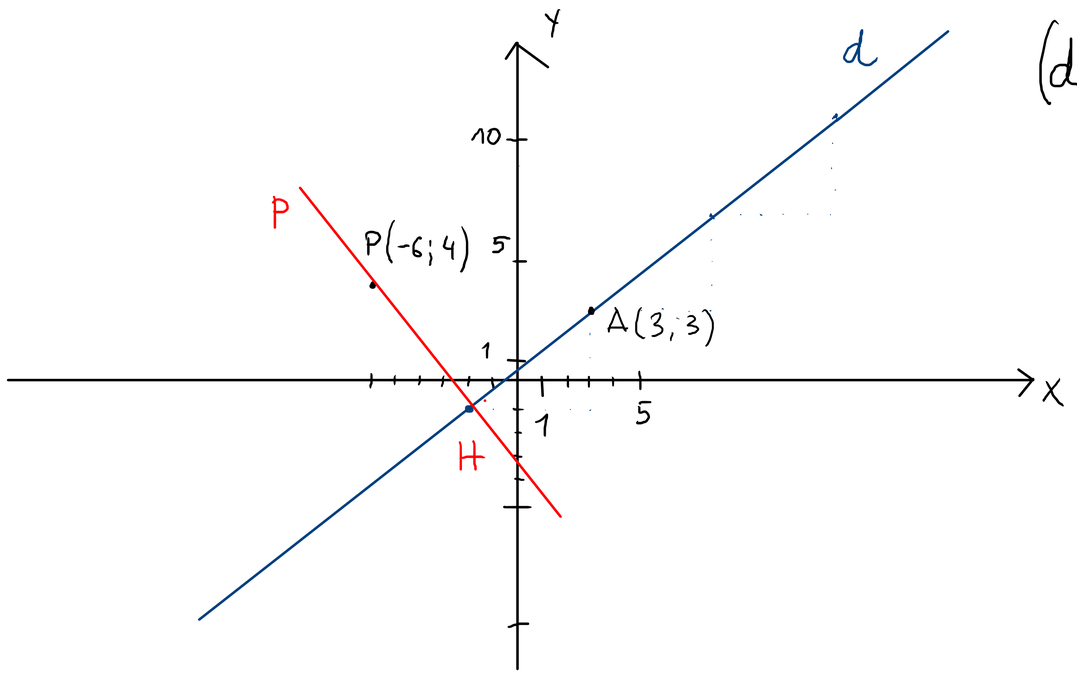
16.04.24

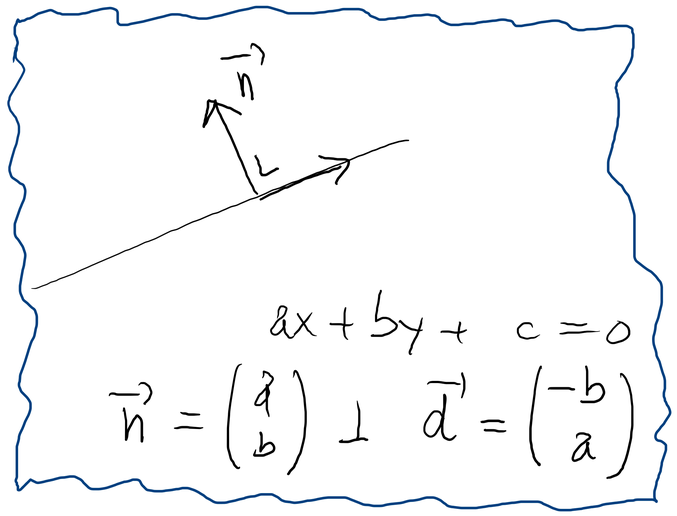
3.1.18 Calculer les coordonnées de la projection orthogonale du point $P(-6;4)$ sur la droite d d'équation $4x = 5y - 3$.



(d) : $4x - 5y + 3 = 0$

$\vec{d} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$, $\vec{n} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$D(-2;-1) \in d$



$ax + by + c = 0$

$\vec{n} = \begin{pmatrix} a \\ b \end{pmatrix} \perp \vec{d} = \begin{pmatrix} -b \\ a \end{pmatrix}$

1) Perpendiculaire p issue de P sur d.

(p) : $5x + 4y + c = 0$

$P \in p$: $5 \cdot (-6) + 4 \cdot 4 + c = 0 \Rightarrow c = 14$

$\boxed{(p) : 5x + 4y + 14 = 0}$

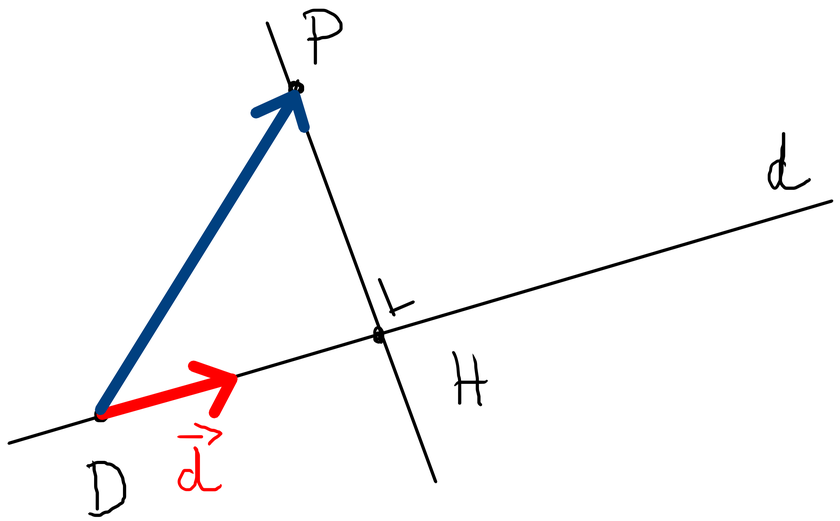
2) Projection de P sur d est l'intersection de d et p.

$$\begin{cases} 5x + 4y = -14 \\ 4x - 5y = -3 \end{cases} \Rightarrow \boxed{H(-2;-1)}$$

$$x = \frac{\begin{vmatrix} -14 & 4 \\ -3 & -5 \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 4 & -5 \end{vmatrix}} = \frac{70 + 12}{-25 - 16} = \frac{82}{-41} = -2$$

$$y = \frac{\begin{vmatrix} 5 & -14 \\ 4 & -3 \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 4 & -5 \end{vmatrix}} = \frac{-15 + 56}{-41} = \frac{41}{-41} = -1$$

2ème méthode



Formulaire

$$\overrightarrow{DH} = \frac{\overrightarrow{DP} \cdot \vec{d}}{\|\vec{d}\|^2} \cdot \vec{d}$$

Donnée : $D(3;3) \in d$ $P(-6;4)$ $\vec{d} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$$\Rightarrow \overrightarrow{DP} = \overrightarrow{OP} - \overrightarrow{OD} = \begin{pmatrix} -6 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$$

$$\overrightarrow{DH} = \frac{-45+4}{41} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = -1 \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$$

$$\overrightarrow{OH} = \overrightarrow{OD} + \overrightarrow{DH} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \Rightarrow H(-2;-1)$$

3.1.19 Calculer les coordonnées du symétrique du point $P(-5;13)$ relativement à la droite $d : 3y + 3 = 2x$.