21.12.23

2.6.12 Calculer la valeur limite de la solution la plus proche de zéro de l'équation $ax^2 + 3x + 1 = 0$ lorsque le coefficient a tend vers 0.

Déterminons les solutions de l'équation si $a \neq 0$.

$$ax^2 + 3x + 1 = 0 \quad , \quad \text{avec } a \neq 0$$

$$\Delta = 9 - 4a$$

$$x_1 = \frac{-3 + \sqrt{9 - 4a}}{2a} \qquad x_2 = \frac{-3 - \sqrt{9 - 4a}}{2a}$$

La solution la plus proche de 0 est x_1 :

$$\lim_{a \to 0} \frac{-3 + \sqrt{9-4a}}{2a} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{Ind}}{=}} \lim_{a \to 0} \frac{-3 + \sqrt{9-4a}}{2a} \cdot \frac{3 + \sqrt{9-4a}}{3 + \sqrt{9-4a}} = \lim_{a \to 0} \frac{9 - 4a - 9}{2a(3 + \sqrt{9-4a})}$$

$$= \lim_{a \to 0} \frac{\overset{-2}{\cancel{-4a}}}{\underset{1}{\cancel{2a}}(3 + \sqrt{9-4a})} = \frac{-2}{6} = -\frac{1}{3}$$

Limites infinies

$$f(x) = \frac{1}{x-1} \quad , \quad ED(f) = \mathbb{R} - \{1\}$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{1}{x-1} \underset{\text{"}\frac{1}{0}\text{"}}{=} \infty \qquad \left\{ \begin{array}{l} \lim\limits_{\substack{x \to 1 \\ <}} f(x) = \underline{-}\infty \\ \lim\limits_{\substack{x \to 1 \\ >}} f(x) = \underline{+}\infty \end{array} \right.$$

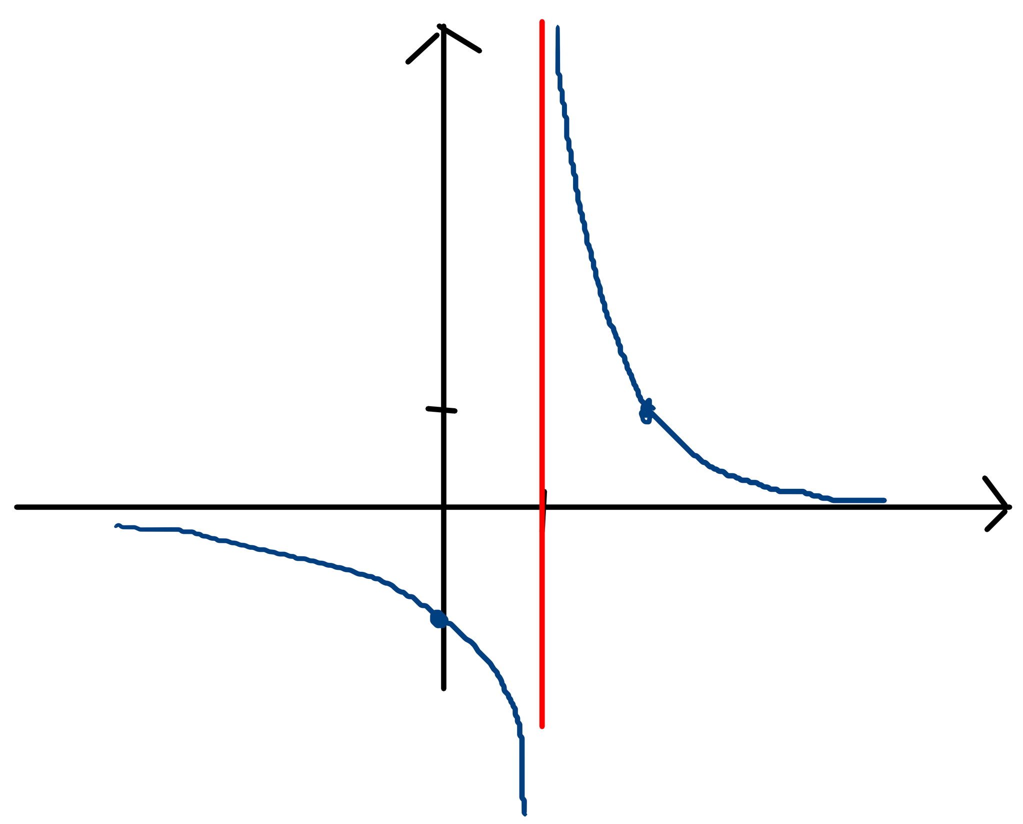
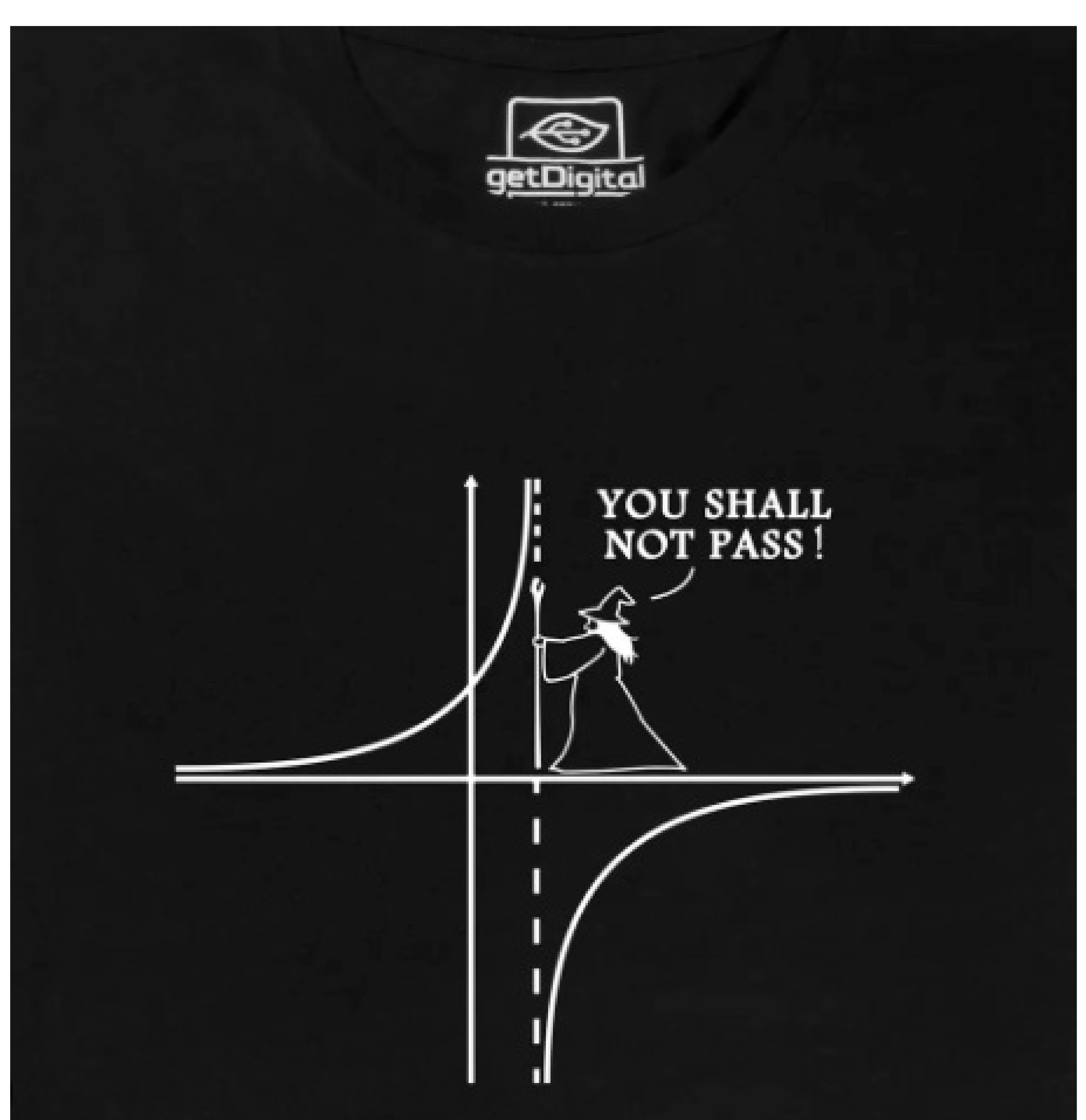


Tableau des signes

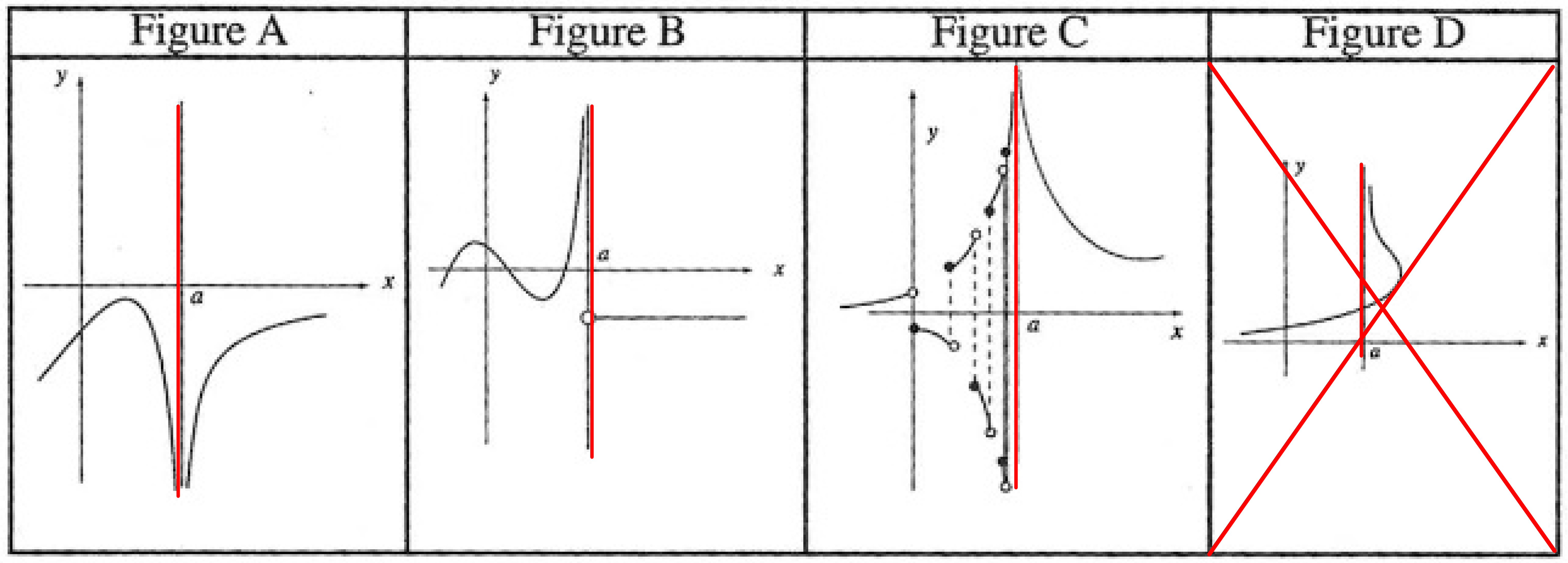
x		1	
$f(x)$	$-$	$-\,\Vert\,+$	$+$

$$f(0{,}99) = \frac{1}{0{,}99-1} = \frac{1}{-0{,}01} = -100$$

$$f(1{,}01) = \frac{1}{1{,}01-1} = \frac{1}{0{,}01} = 100$$



2.6.13 Dire pour chacune des quatre figures ci-dessous quelles sont les notations autorisées parmi 1), 2), . . . , 9) :



pas une fonction !

A) $\lim\limits_{x\to a} f(x) = \begin{cases} 1) & \infty \\ 2) & +\infty \\ 3) & -\infty \end{cases}$ $\quad \lim\limits_{\substack{x\to a \\ <}} f(x) = \begin{cases} 4) & \infty \\ 5) & +\infty \\ 6) & -\infty \end{cases}$ $\quad \lim\limits_{\substack{x\to a \\ >}} f(x) = \begin{cases} 7) & \infty \\ 8) & +\infty \\ 9) & -\infty \end{cases}$

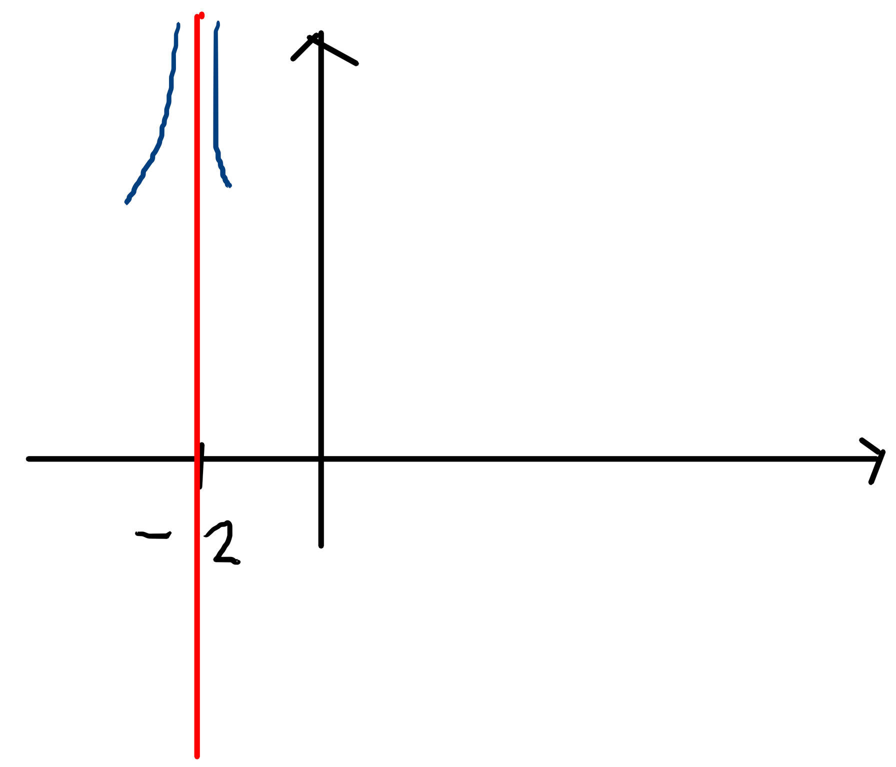
B) $\lim\limits_{x\to a} f(x) = \begin{cases} 1) & \infty \\ 2) & +\infty \\ 3) & -\infty \end{cases}$ $\quad \lim\limits_{\substack{x\to a \\ <}} f(x) = \begin{cases} 4) & \infty \\ 5) & +\infty \\ 6) & -\infty \end{cases}$ $\quad \lim\limits_{\substack{x\to a \\ >}} f(x) = \begin{cases} 7) & \infty \\ 8) & +\infty \\ 9) & -\infty \end{cases}$

C) $\lim\limits_{x\to a} f(x) = \begin{cases} 1) & \infty \\ 2) & +\infty \\ 3) & -\infty \end{cases}$ $\quad \lim\limits_{\substack{x\to a \\ <}} f(x) = \begin{cases} 4) & \infty \\ 5) & +\infty \\ 6) & -\infty \end{cases}$ $\quad \lim\limits_{\substack{x\to a \\ >}} f(x) = \begin{cases} 7) & \infty \\ 8) & +\infty \\ 9) & -\infty \end{cases}$

2.6.14 Calculer les limites suivantes :

a) $\displaystyle\lim_{x\to-2}\frac{x^2+3x+6}{(x+2)^2} = +\infty$

" $\frac{4}{0}$ "



b) $\displaystyle\lim_{x\to-3}\frac{x^2+2x-15}{x^2+8x+15} = \infty$

" $\frac{-12}{0}$ "

$$f(x) = \frac{x^2+2x-15}{x^2+8x+15} = \frac{(x-3)\cancel{(x+5)}}{(x+3)\cancel{(x+5)}} \qquad ED(f) = \mathbb{R} - \{-3; -5\}$$

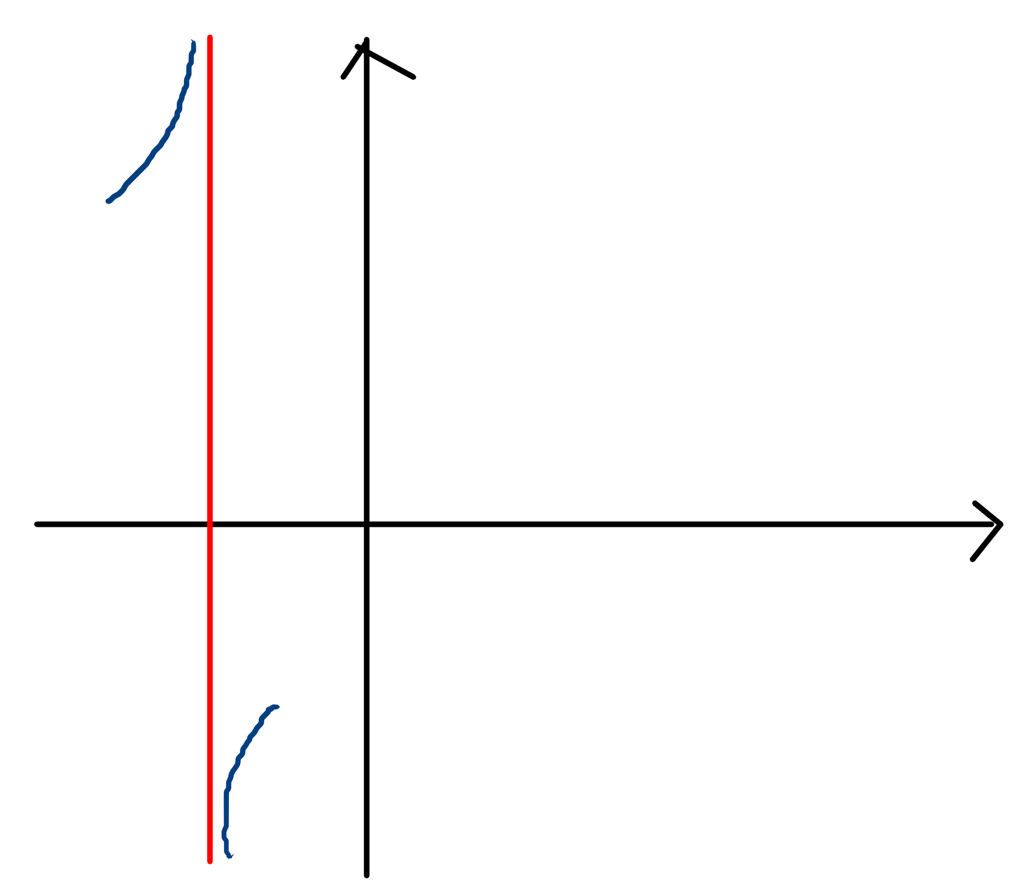
$$= \frac{x-3}{x+3}$$

Signe de $f(x)$:

x		-5		-3		3	
$f(x)$	$+$	$\|\|$	$+$	$\|\|$	$-$	0	$+$

$$\lim_{\substack{x\to-3\\<}} f(x) = +\infty$$

$$\lim_{\substack{x\to-3\\>}} f(x) = -\infty$$



En passant, on voit que $\displaystyle\lim_{x\to-5} f(x) = \frac{-5-3}{-5+3} = \frac{-8}{-2} = 4$

c) $\lim\limits_{x\to 0} \frac{x^2-3x}{x^3} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{Ind}}{=}} \lim\limits_{x\to 0} \frac{\cancel{x}\cdot(x-3)}{\cancel{x}\cdot x^2} = \lim\limits_{x\to 0} \frac{x-3}{x^2} \underset{\text{“}\frac{-3}{0}\text{”}}{=} -\infty$

$$\lim_{x\to 0^{<}} \frac{x^2-3x}{x^3} = \lim_{x\to 0^{<}} \frac{x-3}{x^2} = -\infty$$

$$\lim_{x\to 0^{>}} \frac{x^2-3x}{x^3} = \lim_{x\to 0^{>}} \frac{x-3}{x^2} = -\infty$$

d) $\lim\limits_{\substack{x\to 5\\ >}} \frac{x-3}{5-x} = -\infty$

e) $\lim\limits_{x\to 1}(2x^2-5x+3)\frac{1}{x-1} \underset{\text{“}0\cdot\infty\text{”}}{\overset{\text{Ind}}{=}} \lim\limits_{x\to 1} \frac{2x^2-5x+3}{x-1} = \lim\limits_{x\to 1} \frac{\cancel{(x-1)}(2x-3)}{\cancel{x-1}} = -1$